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Simulation and Analysis of Sandwich Panels Free Vibration with trapezoidal Corrugated Core Based on Galerkin Method

Gholamreza BanadCooki[a*], Masoud MehdizadehRokhi[b]

[a] *PHD Student, Mechanical Engineering Dep., Faculty of Mechanical And Mechatronics Engineering, Shahrood University of Technology, Shahrood, Iran.*

[b] *Assistant professor, Mechanical Engineering Dep., Faculty of Mechanical And Mechatronics Engineering, Shahrood University of Technology, Shahrood , Iran.*

* *Corresponding author e-mail: banadcooki@gmail.com*

Abstract

The purpose of this paper is to evaluate the free vibration of sandwich panels with corrugated core using the element-free Galerkin method and based on the first-order shear deformation theory (FSDT).The sandwich panels' free vibrations with corrugated core consist of two sheets above and below the panels, and a corrugated core in middle of these panels. The core equals to orthotropic sheet and the two panels equal to isotropic sheet. Dynamic equations of the members are obtained through FSDT. The present research applies Galerkin numerical element less method to solve equations of the problems. This method uses the functions of minimum moving squares. The model is simulated in cosmos software; the results are compared with the results of present papers, and show the accuracy of the method applied in the present paper.

Keywords: free vibrations, sandwich structures, corrugated core, Element-free Galerkin methods.

Introduction

Sandwich structures refer to those structures which are composed of two thin skins of high mechanical properties and a thick core yet relatively weak and light; this composition causes the structure to become stronger and heavier. According to the application of these structures, different materials and shapes can be used for the skins and core. For instance, wood, aluminium, plastic, and composite can be used for the skins in flat or corrugated shape; and wood, different types of foam, aluminium, and composite can be used for core in corrugated cores and various honeycomb shapes. Since sandwich composite materials, among other materials used, own such characteristics as low weight while having ideal mechanical properties, excellent resistance against corrosion and chemical agents, relative thermal and sound insulation, and appropriate functionality and stability, they are welcomed by many industries like aerospace, marine, transportation, and construction industries. During the last few years, some studies have conducted on mechanical behaviour of sandwich structures.

Rao studied shear buckling of composite corrugated panel. Sheets waveform is sinusoidal and trapezoidal [1]. He changed the corrugated panel to an equivalent sheet, and obtained its qualities through geometric parameters and panel mechanical properties. The results suggested that in compare to corrugated panel perpendicular to the longer length, the corrugated panel perpendicular to the shorter length had a better function. Using finite element method and analytical solution, Heder dealt with computing buckling load of simple sandwich panels and reinforces sandwich panels [2]. He also calculated buckling load of panels under different boundary conditions by using the method of energy and transformation functions, which satisfy boundary conditions. Applying Rayleigh – Ritz, William and Jackson analysed shear buckling sandwich panel whose cores were made of honeycomb and titanium; and whose skins were composite with metal matrix, and were under the pressure load and shear load on the panel [3]. Yan et al. examined sandwich panel of corrugated cores filled with aluminium foam, which entered the panel from outside of the sheet under pressure load [4]. Using finite element method (FEM) software, they also analysed it. They stated that the panels filled with aluminium are stronger and absorb more energy than foamless corrugated panels filled with aluminium foam. Applying a three-point bending load, Rahmani and Rahimi investigated flexural behaviour of sandwich structures with composite skins and composite cores of foam and several corrugated composite skins [5]. Through simulating the samples in finite element software and comparing them with experimental results, they also expressed that simulating finite element could highly predict the behaviours of these structures under flexible bending load. Zhang et al. evaluated the strength, hardness, and energy absorption of sandwich structures, composite structures, and corrugated structures [6]. They studied effects of the thickness of core and skins as well as wave angel. They suggested that an increase in wave angel and thickness of core developed bending power, while an increase in length of the connected part reduced power. Grenestedt and Reany investigated sandwich panels with one flat skin and one corrugated skin, and a core filled with PVC foam [7]. This research evaluates sandwich panels free vibrations of corrugated cores (CSP) using element-free Galerkin methods and based on basic order shear deformation theory (FSDT).

Moving Least Squares Method

Variable u(x) is considered as displacement field in an elasticity problem, as scalar function, and as the problem unknown; it is defined in the range of Ω. Using the approximation of moving least squares, $u^h(x)$ will be introduced as the following equation:

$$u^h(x)=\sum_{i=1}^{m} p_i(x)a_i(x)=p^T(x)a(x) \qquad (1)$$

Where $u^h(x)$ an approximation of the field variable and u (x) is is in the situation of x.

In above equation, p(x) shows vector of base single sentences (propositions), and m shows the number of these single sentences. These unknown coefficients will calculated so that to minimum the sum of weighted squares error. Therefore, the weighted error second norm will be expressed as the following relation:

$$J=\sum_{i=1}^{n}\omega(x-x_i)\left[P^T(x_i)a(x)-u_i\right]^2 \qquad (2)$$

Where, in mentioned relation, n is the number of nodes within the x local domain and $\omega(x - x_i)$ is the problem weight function, which is appropriate with the distance of x point from the node in the situation of x_i. In addition, in the above equation, u_i shows node parameter of field variable in the situation of node i. the present paper uses quadratic (square) functions $p^T(x) = \left[1, x, y, x^2, xy, y^2\right]^T (n = 6)$ as weighted functions. The unknown coefficients are computed through functional minimizing (2); and another form of this equation is obtained by placing it in (1), that is express as follow based on the functions of moving least squares shape:

$$u^h(x) = \Phi^T(x) U_s \qquad (3)$$

Where the node parameter vector of variable field is, $\Phi(x)$ is shape functions vector of moving least squares, and it is written as below relation:

$$\Phi^T(x) = \left\langle \phi_1(x) \ldots \phi_n(x) \right\rangle = p^T(x).A^{-1}(x).B(x) \qquad (4)$$

In the relation above, matrix A(X), B(X) are defined as following relations:

$$A(x) = \sum_{i=1}^{n} \omega(x - x_i) p(x_i) p^T(x_i) \qquad (5)$$

$$B(x) = \left\langle \omega(x - x_1) p(x_1) \ldots \omega(x - x_n) p(x_n) \right\rangle \qquad (6)$$

Shape functions of the moving least squares, due to nature governing them, are not sensitive towards the number of nodes within local domain. Additionally, accuracy and very high convergence rate with little knots are the advantages offered by these shape functions to their related numerical free element methods.

The Dominant Equations

Figure (1) shows one type of CSP with trapezoidal corrugated cores. which its upper and lower panels are represented by t symbol, and corrugated panel is represented by p. Furthermore, Cartesian coordinates (x,y,z) are applied to describe the geometry and dimensions of panels. (Figure 2)

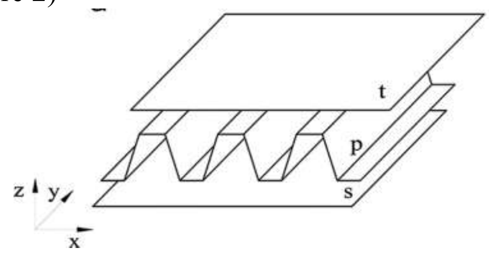
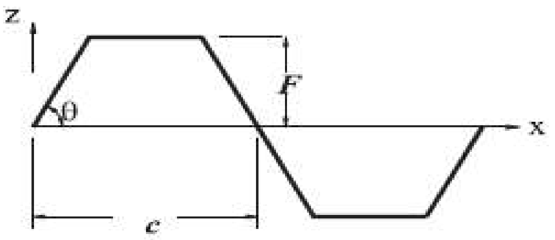


Figure 1. Two types of CSP with trapezoidal corrugated cores



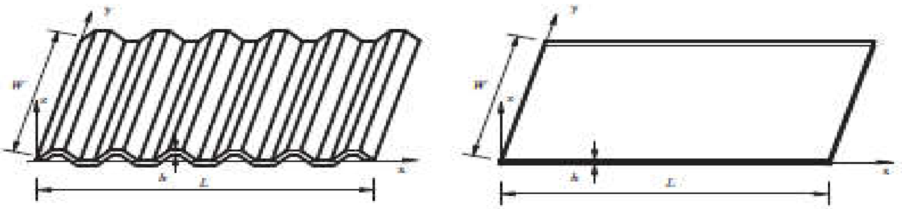


Figure 2.CSP in Cartesian coordinate's panel and a wave from trapezoidal geometry

Stress and strain relationships will be (7) and (8) equations:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\frac{1}{(1-\mu_x\mu_y)}\begin{bmatrix}E_x & E_\mu & 0\\ E_\mu & E_y & 0\\ 0 & 0 & (1-\mu_x\mu_y)G_{xy}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \tag{7}$$

$$\begin{bmatrix}\tau_{xz}\\ \tau_{yz}\end{bmatrix}=\begin{bmatrix}G_{xz} & 0\\ 0 & G_{yz}\end{bmatrix}\begin{bmatrix}\gamma_{xz}\\ \gamma_{yz}\end{bmatrix} \tag{9}$$

Where G_{yz} ، G_{xz} ، G_{xy} ، μ_y ، μ_x ، E_μ ، E_y ، E_x ،

Are elastic properties given in paper [11,13]. Elastic properties trapezoidal will as equation (9):

$$\begin{aligned}
&\mu_y=\mu,\\
&\mu_x=\frac{c^2h_p^2}{clh_p^2+12\alpha(1-\mu^2)}\mu,\\
&\alpha=\frac{F^3}{3\tan\theta}+F^2bw+\frac{1}{3}\tan^2\theta(c^3-(bw+F/\tan\theta)^3)-(2F+bw\tan\theta)\tan\theta(c^2-(bw+F/\tan\theta)^2)\\
&+(2F+bw\tan\theta)^2(c-bw+F/\tan\theta)\\
&E_x=\frac{E(1-\mu_x\mu_y)c}{l(1-\mu^2)},\\
&E_y=\frac{\mu_y}{\mu_x}E_x
\end{aligned} \tag{9}$$

And also $G_{xy}=G_{xz}=G_{yz}=E/2(1+\mu)$. According to the theory, first order shear deformation of displacement field [14] will be as follow:

$$\begin{bmatrix}w\\ u\\ v\end{bmatrix}=\sum_{I=1}^{N}\begin{bmatrix}N_I(x,y) & 0 & 0\\ 0 & zN_I(x,y) & 0\\ 0 & 0 & zN_I(x,y)\end{bmatrix}\begin{bmatrix}w_I(t)\\ \varphi_{xI}(t)\\ \varphi_{yI}(t)\end{bmatrix} \tag{10}$$

Where, $[w_I(t),\varphi_{xI}(t),\varphi_{yI}(t)]^T=\delta_I$ are nodes parameter in i node and represent time t. Curvature of the middle plate as well as bending and shear strain orthotropic plate are shown as the below relations:

$$\begin{aligned}
&\varepsilon=\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}=\begin{bmatrix}\varphi_{x,x}\\ \varphi_{y,y}\\ \varphi_{x,y}+\varphi_{y,x}\end{bmatrix}=\sum_{I=1}^{n}B_{bI}\delta_I\\
&\gamma=\begin{bmatrix}\gamma_{xz}\\ \gamma_{yz}\end{bmatrix}=\begin{bmatrix}w_{,x}+\varphi_x\\ w_{,y}+\varphi_y\end{bmatrix}=\sum_{I=1}^{n}B_{sI}\delta_I
\end{aligned} \tag{11}$$

in which

$$B_{bI}=\begin{bmatrix}0 & N_{I,x} & 0\\ 0 & 0 & N_{I,y}\\ 0 & N_{I,y} & N_{I,x}\end{bmatrix},B_{sI}=\begin{bmatrix}N_{I,x} & N_I & 0\\ N_{I,y} & 0 & N_I\end{bmatrix} \tag{12}$$

Using Hamilton principle for free vibration analysis and with no damper, equation [13] is obtained:

$$\delta\int_{t_1}^{t_2}(T-\Pi)dt=0 \tag{13}$$

So, linear elastic strain energy II and kinetic energy T are respectively expressed by the relations (14) and (15) in an integral form:

$$\Pi=\frac{1}{2}\iint\int_{-h/2}^{h/2}\varepsilon^T D\varepsilon dzdxdy+\frac{1}{2}\iint\gamma^T A_s\gamma dxdy \tag{14}$$

$$T=\frac{1}{2}\iint\int_{-h/2}^{h/2}\dot{U}^T\rho\dot{U}dzdxdy \tag{15}$$

Where we have:

$$D=\frac{h^3}{12(1-\mu_x\mu_y)}\begin{bmatrix}E_X & E_\mu & 0\\ E_\mu & E_y & 0\\ 0 & 0 & (1-\mu_x\mu_y)G_{xy}\end{bmatrix},A_s=\frac{h}{k}\begin{bmatrix}G_{xz} & 0\\ 0 & G_{yz}\end{bmatrix} \tag{16}$$

Placing (10 – 12) and (19 -20) in (13), we will have:

$$K\delta+M\ddot{\delta}=0 \tag{17}$$

Through some computations and by using the principle of sum of effects (superposition) for K, M andwe have:, δ

$$K=\begin{bmatrix}K_{ii}^i & K_{ib}^t & 0 & 0 & 0\\ K_{bi}^t & K_{bb}^t+K_{11}^p & K_{12}^p & K_{13}^p & 0\\ 0 & K_{21}^p & K_{22}^p & K_{23}^P & 0\\ 0 & K_{31}^p & K_{32}^p & K_{33}^p+K_{bb}^s & K_{bi}^s\\ 0 & 0 & 0 & K_{ib}^s & K_{ii}^s\end{bmatrix},$$

$$M=\begin{bmatrix}M_{tt}^i & M_{ib}^t & 0 & 0 & 0\\ M_{bi}^t & M_{bb}^t+M_{11}^p & M_{12}^p & M_{13}^p & 0\\ 0 & M_{21}^p & M_{22}^p & M_{23}^P & 0\\ 0 & M_{31}^p & M_{32}^p & M_{33}^p+M_{bb}^s & M_{bi}^s\\ 0 & 0 & 0 & M_{ib}^s & M_{ii}^s\end{bmatrix},\delta=\begin{bmatrix}\delta_i^t\\ \delta_b^t\\ \delta_2^p\\ \delta_b^s\\ \delta_i^s\end{bmatrix} \tag{18}$$

Solving the equation of the standard special relationship (19), vibrations frequency CSP will be achieved:

$$(K+\omega^2 M)\delta=0 \tag{19}$$

Numerical Simulation and the Results

According to figure (2), the dimensions of the model are L=1.2m, W=1.2m, F= 0.015, h=0.02m, c=0.1m, and angel θ=45°.

Here, the degree indicates that the panel has 6 corrugations (h is thickness; and the definitions for F and c are given in figure 2), Young's modulus sheet equals

$E = 3x10^7$ Pa and Poisson's ratio (coefficient) is $\mu=0.3$, and density is $\rho = 1000$ kg/m 3.

The above-mentioned ten CSP's vibration mode, with different support conditions are:

1. CCCC: we study all clamped supports.
2. SSSS: we study all simple supports.
3. CCSS: We study clamped support along the straight side and simply support along the corrugated side.
4. SSCC: We study simply support along the straight side and clamped support along the corrugated side.
5. CCFF: We study clamped support along the straight side and free support along the corrugated side.
6. FFCC: We study free support along the straight side and clamped support along the corrugated side.

Figure (3) shows sandwich panel with trapezoidal corrugated cores. In addition, vibrations frequency along with ten first form constructs mode CSP (figure 4) are respectively extracted as follow; and software shell mode of Cosmos finite element (COSMOS WORK Ver.2014 sp4) are applied to compare numerical results with analytical results. The results are represented as the following diagrams and figures.

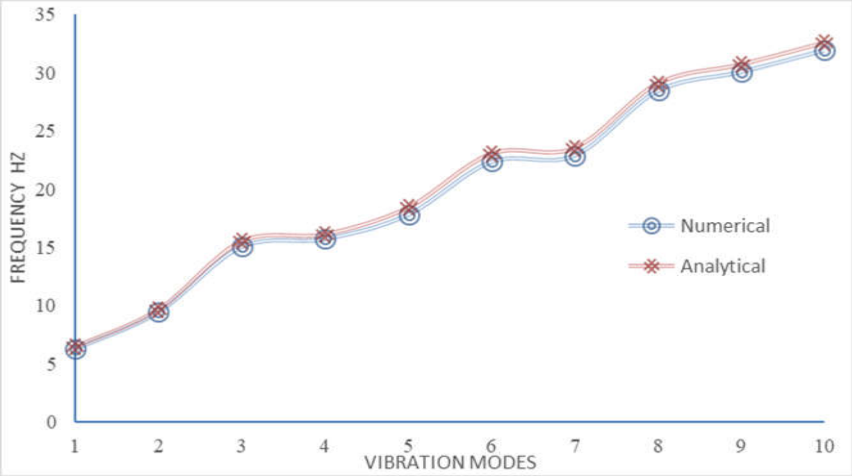


Diagram 1. Vibration frequencies related to ten primary structural modes CSP of trapezoidal corrugated core with boundary conditions CCCC

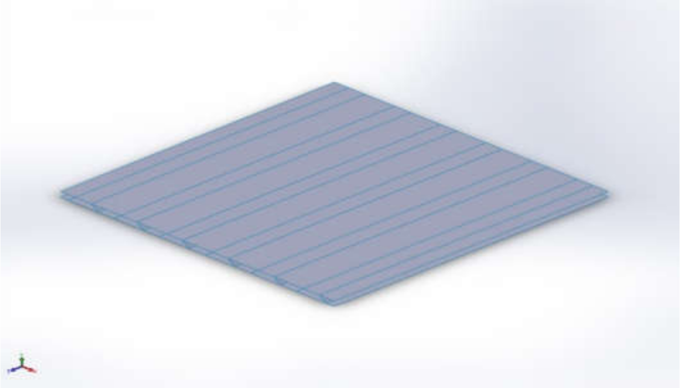


Figure 3. Panel sandwich with trapezoidal corrugated cores

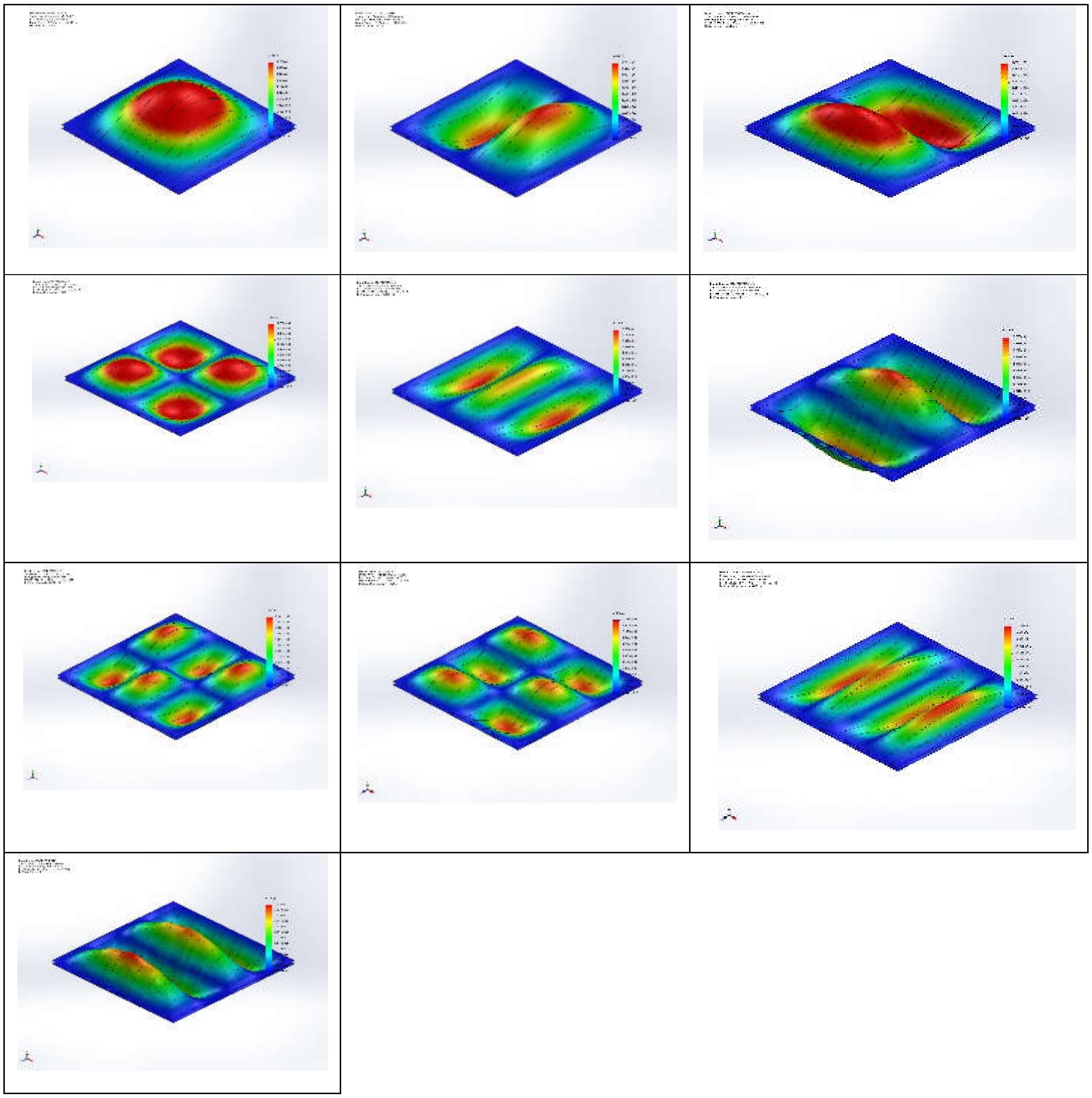


Figure 4. Forms of ten basic structural modes CSP with trapezoidal corrugated cores

According to diagram 2, all simple support and vibrations frequencies are extracted respectively for ten basic structural forms CSP, and its results are shown in the diagram below:

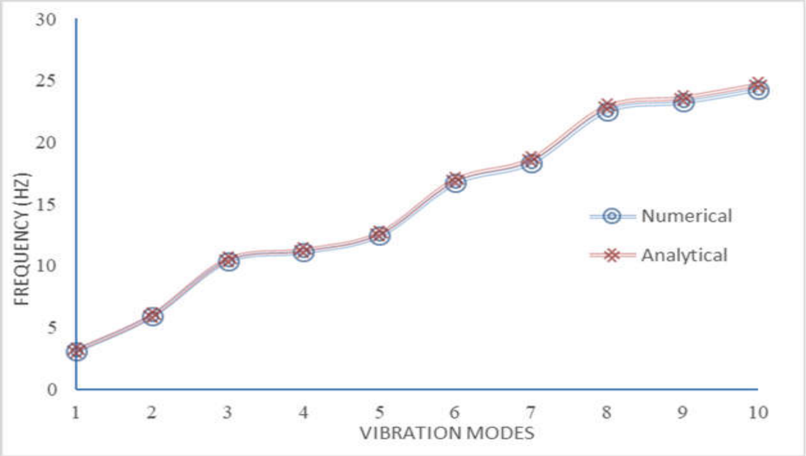


Diagram 2. The vibration frequencies related to ten basic structural modes CSP of trapezoidal corrugated core with boundary conditions SSSS

According to diagram 3, two clamped supports, two simple supports, and vibration frequencies and vibration frequencies for ten basic structural mode CSP are extracted respectively, and the results are represented as the diagram below:

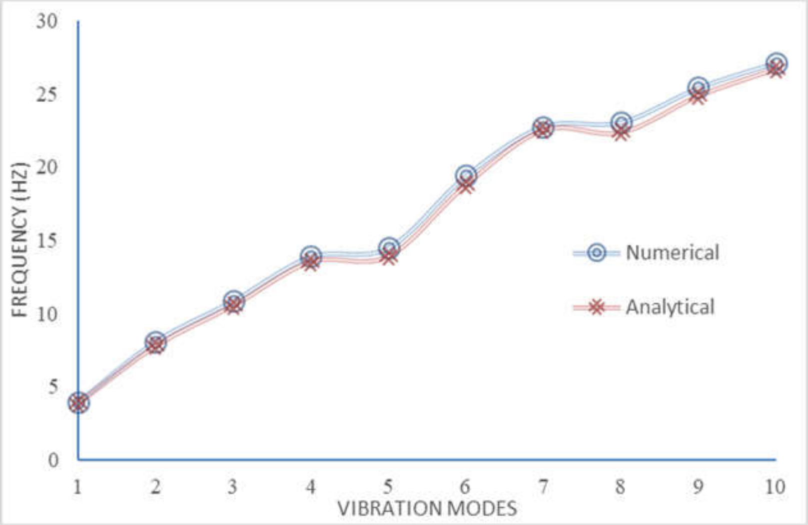


Diagram 3. The vibration frequencies related to ten basic structural modes CSP of trapezoidal corrugated core with boundary conditions CCSS

Based on diagram 4, two simple supports, two clamped supports, and vibration frequencies for ten basic structural mode CSP are extracted respectively, and the results are represented as the diagram below:

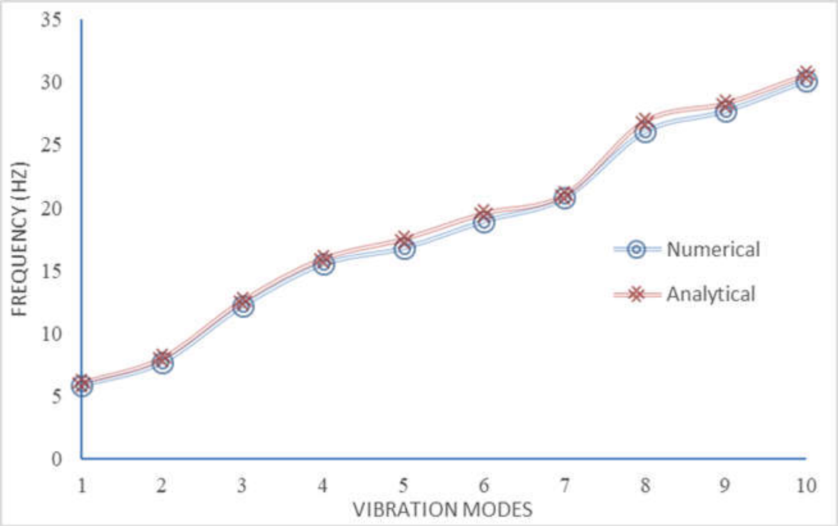


Diagram 4. The vibration frequencies related to ten basic structural modes CSP of trapezoidal corrugated core with boundary conditions SSCC

According to diagram 5, two clamped supports, two free supports, and vibration frequencies and vibration frequencies for ten basic structural modes CSP are extracted respectively, and the results are represented as the diagram below:

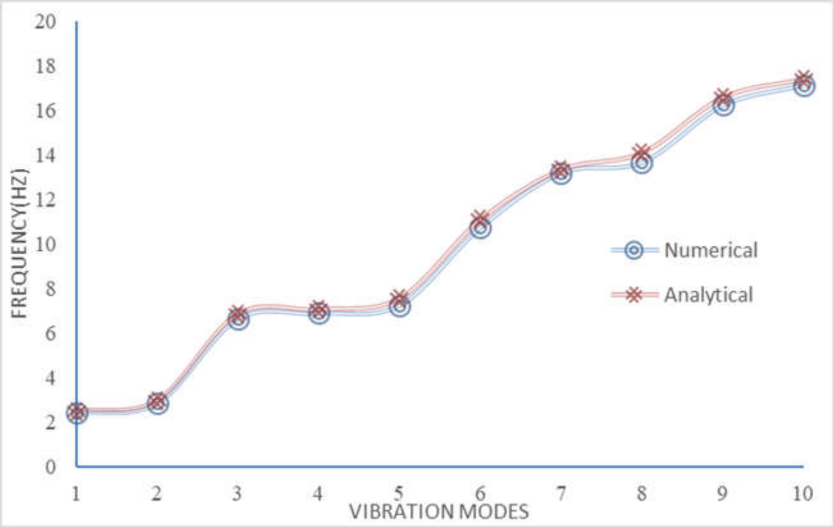


Diagram 5. The vibration frequencies related to ten basic structural modes CSP of trapezoidal corrugated core with boundary conditions CCFF

According to diagram 6, two free supports, two clamped supports, and vibration frequencies and vibration frequencies for ten basic structural mode CSP are extracted respectively, and the results are represented as the diagram below:

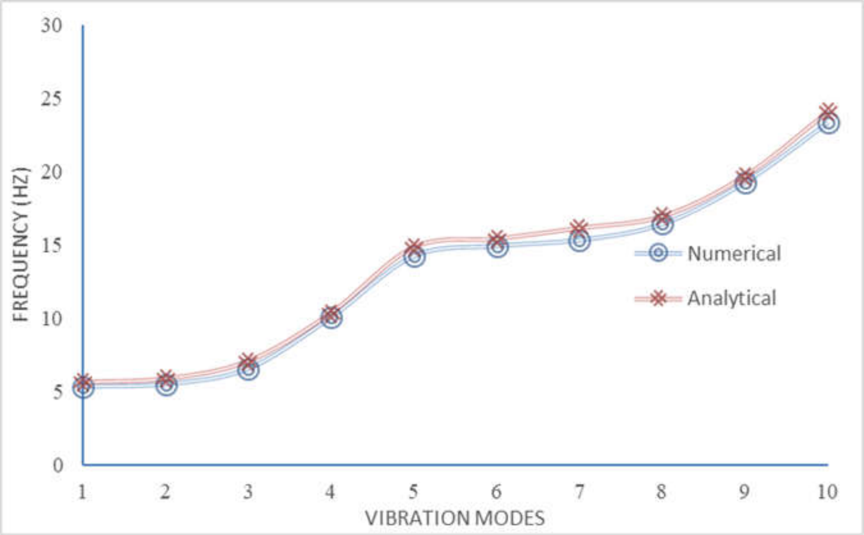


Diagram 6. The vibration frequencies related to ten basic structural modes CSP of trapezoidal corrugated core with boundary conditions FFCC

Conclusion

The above diagrams indicate natural frequencies Curve in terms of vibration modes for CSP structure. In addition, the accuracy of Galerkin free element method for analysing vibrations is compared with its analytical and numerical solution. The present paper deals with Galerkin free element method for analysing free vibrations of panels' sandwich with core. A CSP structure is a composite structure which is made of three parts including two panels and a corrugated core. The corrugated core is estimated by an orthotropic sheet. CSP dynamic equations are made of dominant superposition equation of these three parts. The given method is the basis for solving CSP future problems such as high-amplitude vibrations while by using the traditional methods, the limited parts need to take more time for mesh-making, and this leads to solution with low accuracy. The approximation of corrugated cores with an orthotropic plate simplified the analysis. Using Cosmos software, the introduced methods were analysed and compared through some examples.

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