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Investigating Earthquakes Effect on the Dynamic Parameters of Highrise Structure using Autoregressive Moving Average with Exogenous Input Model

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Abstract

Large-scale civil structures, such as high-rise structures, are exposed to environmental loads and earthquake excitations. On the other hand, considering the great importance of these structures in modern urban societies, it is vital to know the behaviour of the structure during an earthquake. Earthquake excitation often affects structures from two directions, as a result, the system identification problem, in this case, is a complex multi-input multi-output problem. This study investigates a parametric time domain method to reduce the negative impact of these issues. In this regard, an AutoRegressive Moving Average with eXogenous input (ARMAX) was validated on the response signals of a numerical model, which indicates the high accuracy of the proposed algorithm for identifying the modal characteristics of the structure under earthquake excitation. And in the next step, the responses of a 72-story tall concrete structure recorded by accelerometer sensors were used to investigate the seismic effects on the dynamic characteristics of the real structures. The results obtained from the proposed method have been compared with the results obtained from the frequency domain decomposition (FDD) method applied on ambient vibration data. The results suggest that the dynamic characteristics under earthquake and ambient excitation are different from each other, which should be considered in vibration-based monitoring.

Keywords: System identification; Modal analysis; Highrise structure; ARMAX.

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1. Introduction

Progressive wear and tear in civil structures due to aging under the influence of environmental conditions has become a global concern. The structural health monitoring (SHM) approach can be used to detect the damage pattern and deal with hostile phenomena. In fact, one of the main advantages of structural health monitoring is structural safety management and making it possible to detect deterioration and damage in the early stages to prevent catastrophic failure. Recently, the rapid progress in high-speed computers and smart sensor technology has enabled the use of computationally complex algorithms and dense arrays of sensors in the field of SHM. In this regard, the measured input/output or output-only data are used to extract the dynamic parameters of the structure such as natural frequencies and mode shapes. To date, several methods have been introduced to determine structural modal parameters in the frequency or time domain, including Eigensystem realization algorithm (ERA), stochastic subspace identification (SSI), prediction error method (PEM), Hilbert transforms and modified frequency domain decomposition (rFDD).

System identification techniques are generally divided into two categories: methods based on input-output data and methods that use only output data. Typically, an input-output technique is more accurate, but input data is not always available. In civil engineering structures, the inputs and outputs are the time records of sensors that are installed in different places of the structure. In the case that the response of the structure is taken under seismic excitation, the histories recorded in the basement and floors of the building are used as input and output signals, respectively [1].

In the last two decades, many parametric techniques based on input-output data that use iterative and recursive algorithms have been introduced. However, system identification methods with real earthquake input have been scarce. In their research, Li and Mau used extended least-square-output-error method to identify a 15-story building under the Whittier earthquake [2]. Loh and Lin identified modal parameters of the seven-story concrete building under several earthquake records. They used the autoregressive model with external input (ARX) [3]. Smyth et al. used a combination of linear and nonlinear detection methods on the Thomas Vincent Bridge in Los Angeles to obtain a complete reduced-order dynamic model [4]. Arici and Mosalam investigated the autoregressive model with external input (ARX) on seven different bridges [5]. Due to the inevitable noises in the recorded data, the order of the ARX model is usually chosen higher than the actual structure. Therefore, various methods have been introduced to filter noisy modes [6].

Li et al used the ARX-ERA method. As mentioned, if there is noise, the order of the ARX model is considered higher than the actual model. Therefore, the fitted model may contain more poles and zeros in the transfer function than the actual structure. This leads to the identification of a large number of noisy modes [7]. Because of this limitation, they proposed to extract the system impulse response from the ARX model and use it as input for the ERA method. The authors believe that this helps to separate the real modes of the structure from the noisy modes. Another limitation of the ARX model is that the poles of the stochastic part of the transfer function are the same as the poles of the deterministic part of the transfer function. This coupling of poles is not realistic because the dynamics of deterministic and stochastic parts may not necessarily have the same poles. This limitation will be more severe when the input vibration includes colored noise, which leads to bias in the results [8]. Therefore, to overcome the mentioned drawbacks, in the present study, the ARMAX model is used to consider the disturbance dynamics with the minimum degree of the model and identify the dynamic parameters of the high-order structure more precisely.

2. Modeling with deterministic input and ARMAX model

The autoregressive moving average model with external input (ARMAX) is broadly used in literature to model the time-varying linear dynamic system that is stimulated by a specified input. Fig. 1 shows the diagram of ARMAX model.

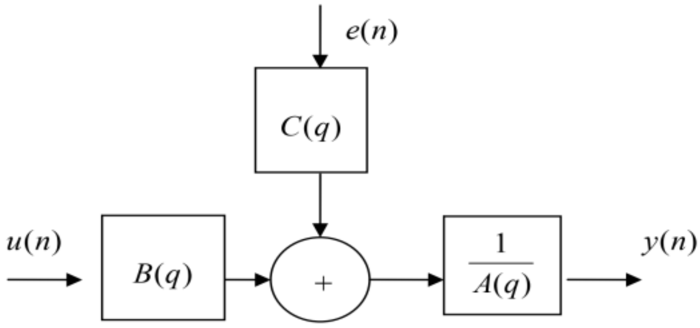

Figure 1. ARMAX model diagram.

ARMAX model can be expressed as Eq. (1).

$$\begin{aligned} y(t)+a_1 y(t-1)+\ldots+a_{n_a} y\left(t-n_a\right)=b_1 u\left(t-n_k\right)+\ldots+b_{n_b} u\left(t-n_k-n_b+1\right) \\ +c_1 e(t-1)+\ldots+c_{n_c} e\left(t-n_c\right)+e(t) \end{aligned} \tag{1}$$

Where a_i are output coefficients and b_i are input coefficients. c_i are the coefficients of the stochastic part, n_k is the delay between the input and output of the system, n_a, n_b and n_c are the orders of the model that must be chosen carefully, and $e(t)$ is the term of white noise that indicates the disturbance in the system.

Due to the fact that the ARMAX model does not have an analytical solution, in order to accurately estimate the optimal ARMAX model, repetition should be used and continued until approach to the appropriate convergence. It is also possible that the ARMAX model gets stuck in a local minimum point, in which case starting from different positions is effective in identifying the model [8]. The appropriate method to identify the ARMAX structure, which is also used in this study, is to use the prediction error method (PEM) using MATLAB.

2.1 Estimating the dynamic characteristics of the structure

After obtaining the unknown parameters of the identification models, the AR part of the model and the state space matrices A and C of the system are obtained according to Eq. (2).

$$\begin{gathered} \hat{y}(k)=-A_1 y(k-1)-A_2 y(k-2)-\ldots-A_{na} y(k-na) \\ \mathbf{A}=\begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & I \\ 0 & I & 0 & \cdots & 0 \\ -A_{na} & -A_{na-1} & -A_{na-2} & \cdots & -A_1 \end{bmatrix}, \quad C=\begin{bmatrix} I & 0 & \cdots & 0 \end{bmatrix} \end{gathered} \tag{2}$$

Where I is the Identity matrix and A_i are the parameters identified in the AR section of the model, which are scalar in single-output cases and square matrices in the multi-output cases whose dimensions are equal to the number of system outputs.

Next, we decompose the **A** matrix in Eq. (2) as Eq. (3).

$$\mathbf{A}=\mathbf{\Psi}\boldsymbol{\lambda}\mathbf{\Psi}^{-1}, \quad \boldsymbol{\lambda}=\operatorname{diag}\left\{\lambda_i\right\} \tag{3}$$

In the above relation $\mathbf{\Psi}$ is the matrix of eigenvectors of the **A** matrix and the diagonal of matrix $\boldsymbol{\lambda}$ includes the eigenvalues of the matrix **A**. finally, the frequencies and mode shapes of the structure are calculated from Eq. (4).

$$f_i=\frac{\left|\ln\left(\lambda_i\right)\right|}{2\pi}\cdot f_s \quad (Hz) \quad , \quad \phi=\mathbf{C}\mathbf{\Psi} \tag{4}$$

2.2 Modal Assurance Criterion (MAC)

One of the most popular tools for quantitatively comparing the mode shapes is the Modal Confidence Criterion. MAC is a statistical index that is based on linear regression and least squares and provides an index to evaluate the compatibility and difference between two mode shapes, which is expressed as Eq. (5) [9].

$$\mathrm{MAC}(r,q)=\frac{\left|\{\varphi_A\}_r^T\{\varphi_X\}_q\right|^2}{\left(\{\varphi_A\}_r^T\{\varphi_A\}_r\right)\left(\{\varphi_X\}_q^T\{\varphi_X\}_q\right)} \tag{5}$$

To evaluate by MAC, the mode shapes are first calculated by different methods [9]. In the above relationship, $\{\varphi_A\}_r$ is r^{th} mode shape by method A, and $\{\varphi_B\}_q$ is q^{th} mode shape by method B. If the mode shapes are calculated by only one method, the obtained parameter is called AutoMAC. MAC matrix values are between 0 and 1. If the two modes are very similar to each other, the correspondence MAC value is calculated close to 1, and if the two modes are not similar, its value is close to zero.

3. Numerical model

The introduced method is applied on a simulated model. The simulated model is a two-dimensional four-story frame according to Fig. 2, which is modeled in the form of state space in MATLAB.

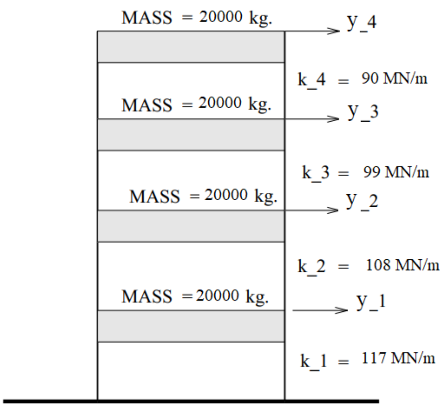

Figure 2. Four degrees of freedom frame system.

The equation of motion of the system for the case of vibration under an earthquake will be Eq. (6).

$$\mathbf{m}\ddot{\mathbf{u}}+\mathbf{c}\dot{\mathbf{u}}+\mathbf{k}\mathbf{u}=\mathbf{p}_{\text{eff}}(t) \quad , \quad \mathbf{p}_{\text{eff}}(t)=-\mathbf{m}\{\iota\}\ddot{u}_g(t) \tag{6}$$

Where **m**, **c**, **k** and ι are the matrices of mass, damping, stiffness and unit vector with size equal to degrees of freedom, respectively.

To be more similar to reality, the damping of the system is also considered and the damping matrix is calculated according to the mass and stiffness matrices according to Eq. (7) [10].

$$\mathbf{C}_{damp}=a_0\mathbf{m}+a_1\mathbf{k} \quad , \quad \frac{1}{2}\begin{bmatrix}1/\omega_i & \omega_i\\ 1/\omega_j & \omega_j\end{bmatrix}\begin{Bmatrix}a_0\\ a_1\end{Bmatrix}=\begin{Bmatrix}\zeta_i\\ \zeta_j\end{Bmatrix} \tag{7}$$

In Eq. (7), the first and second modes are used to calculate a_0 and a_1 coefficients, and the damping ratio for both modes is assumed to be 3%.

Finally, the state space model has been used for simulation according to relations 8 and 9.

$$\dot{\mathbf{x}}=\mathbf{A}\,\mathbf{x}+\mathbf{B}\,\mathbf{u} \quad , \quad \mathbf{A}=\begin{bmatrix}\mathbf{0}_{n\times n} & \mathbf{I}_{n\times n}\\ \left[-\mathbf{M}^{-1}\mathbf{K}\right]_{n\times n} & \left[-\mathbf{M}^{-1}\mathbf{C}_{damp}\right]_{n\times n}\end{bmatrix} \quad , \mathbf{B}=\begin{bmatrix}\mathbf{0}_{n\times n}\\ \left[\mathbf{M}^{-1}\right]\end{bmatrix} \tag{8}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \quad , \quad \mathbf{C} = \left[\left[-\mathbf{M}^{-1}\mathbf{K} \right]_{n\times n} \quad \left[-\mathbf{M}^{-1}\mathbf{C}_{damp} \right]_{n\times n} \right] \;,\; \mathbf{D} = \left[\mathbf{M}^{-1}_{n\times n} \right] \tag{9}$$

Eq. (8) is known as the state equation, where **x**, **A**, **B**, and **u** are the state vector, state matrix, input matrix, and input of the system, respectively. Eq. (9) is also known as the observation or output equation, where **x** and **u** are similar to Eq. (8), and **y**, **C**, **D** are the output, output matrix, and direct transition, respectively. **C** and **D** matrices do not have a single relationship and are different depending on which parameter is chosen for the output vector. If the output vector includes the acceleration of each degree of freedom, **C** and **D** can be calculated from Eq. (9).

The input of the system in Fig. 2 is an earthquake record that was entered at the base of the structure and the outputs of the structure were simulated. Finally, to consider the measurement errors, 10% white noise with uniform distribution up to 50 Hz range was added to the simulated outputs. The results for this condition are shown along with analytical mode shapes in Fig. 3 and Table 1. According to the obtained results, it can be seen that the identified and analytical mode shapes match each other with great precision.

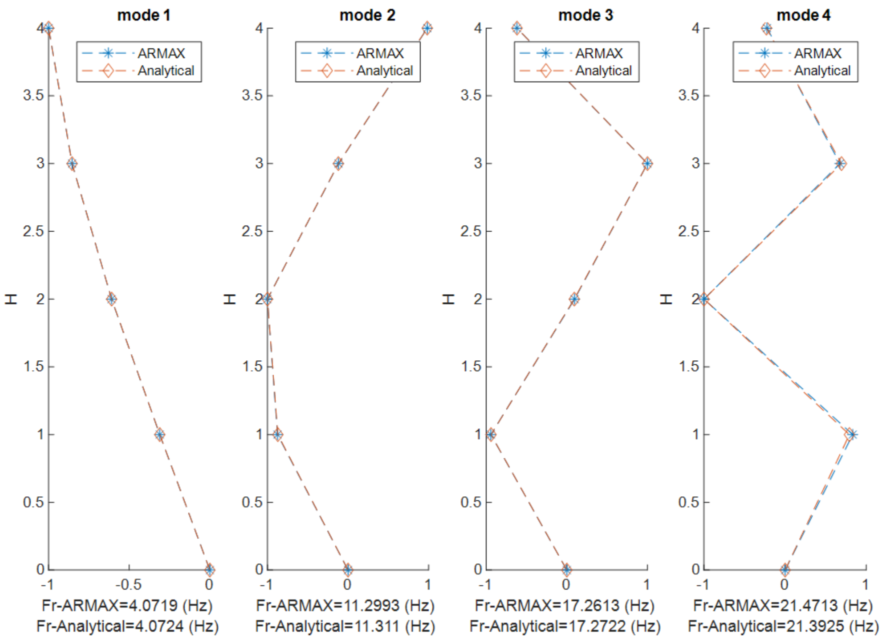

Figure 3. Frequencies and mode shapes identified by the ARMAX method along with the frequencies and mode shapes of the analytical model.

Table 1. Identified and analytical frequencies.

Mode	Analytical	ARMAX	Error(%)
1	4.072	4.072	0
2	11.311	11.299	0.106
3	17.272	17.261	0.064
4	21.393	21.471	0.365

4. Implementation of the method on a tall structure

The Wilshire Grand Center, shown in Fig. 4-(a), is a 335.3 m skyscraper in the financial district of downtown Los Angeles. This building, which was completed in 2017, is the tallest building in the western United States. The building is part of a mixed-use hotel, retail, shopping center, and office complex. The structure is a 72-story structure that has accelerometer sensors embedded in its different levels according to Fig. 4-b. The structure has experienced various earthquakes and its dynamic response has been recorded by sensors. The last earthquake that is also used in this study is described in Table 2. The mentioned structure was identified using the ARMAX method in the east-west direction (section A-A) shown in Fig. 4-(b), and its results are as shown in Fig. 7-6.

Table 2. Characteristics of the employed earthquake.

Station	Epicentral Distance (km)	Ground motion pga (g)	Floor Acc. (g)	Earthquake Name	Magnitude
Wilshire Grand Center	16.6	0.024	0.149	SouthElMonte	4.5MW

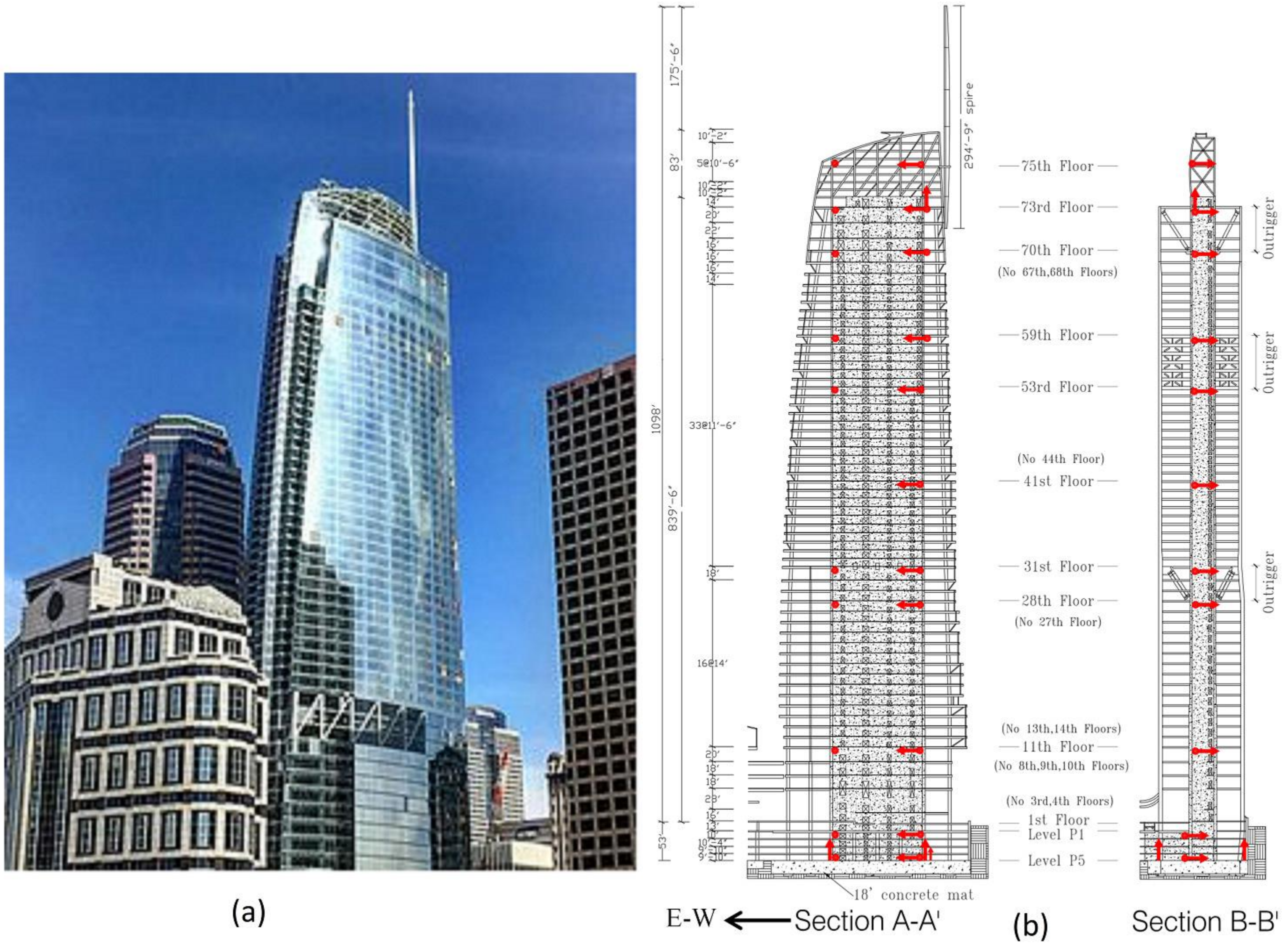

Figure 4. View of the structure(a), and sensors position(b).

One of the modal identification methods that is well known for ambient vibrations is the frequency domain decomposition (FDD) method, but its accuracy is not optimal for the vibration under earthquake excitation. As the responses of the structure under ambient vibration are also recorded, five modes of the structure were identified using FDD method, and the frequencies of each of them are shown in Fig. 5-(a). These ambient vibrations' output can be considered as a benchmark to compare the results. Fig 5-(b) and Table 3 show the mode shapes and frequencies obtained from the two methods side by side. It can be seen that the frequencies obtained by the ARMAX method in the earthquake input case are generally lower than the frequencies of the FDD method, and this difference is even more for the first modes. since the structure is equipped with some special seismic bearing system such as buckling-restrained braces (BRB) and Outrigger system, the difference in frequencies in the ambient vibration and the seismic vibration cases can be related to this type of equipment, which purportedly has its effects on higher vibration intensities. The difference in frequencies in different states of excitation shows the importance of using appropriate identification methods in the earthquake input scenario. The MAC was also calculated between the modes obtained from the two methods, the result of which can be seen in Fig. 6. As can be seen, the diagonal elements of the MAC matrix are very close to one, which shows that the modes obtained from The ARMAX and the FDD method are consistent in total.

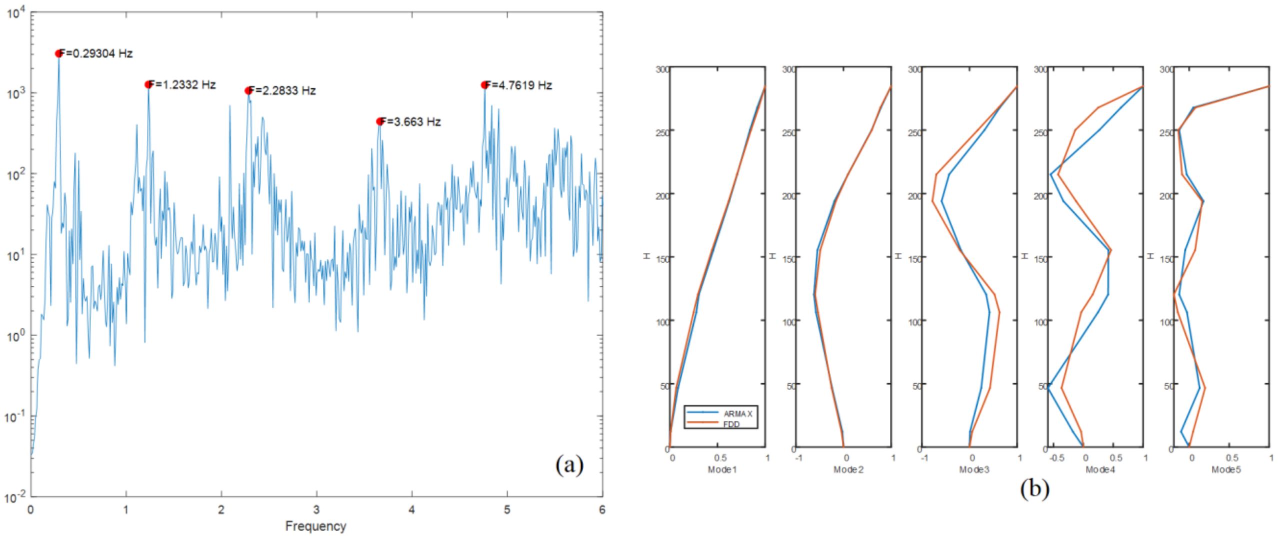

Figure 5. Frequency peaks in the FDD method (a). Identified mode Shapes by the two methods (b).

Table 3. Identified frequencies using the two methods.

Mode	ARMAX	FDD
1	0.234	0.293
2	1.141	1.233
3	2.288	2.283
4	3.524	3.663
5	4.869	4.762

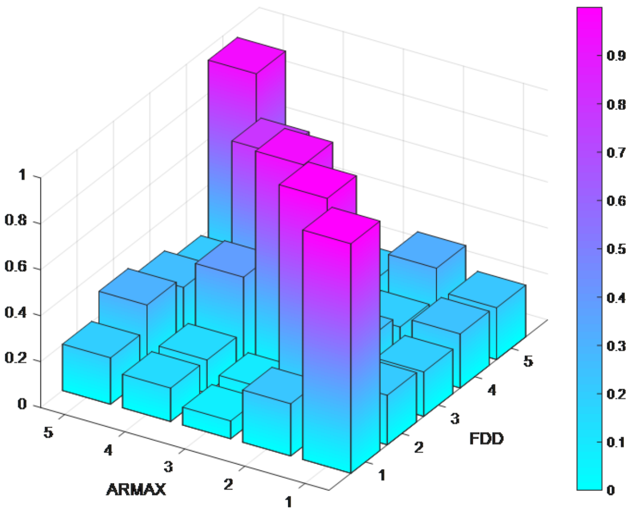

Figure 6. MAC matrix calculated using mode shapes obtained from ARMAX and FDD.

5. Conclusion

In this article, the extraction of dynamic parameters including natural frequencies and vibration mode shapes of a real high-rise structure was investigated using data collected by accelerometer sensors during an earthquake event. In this regard, the autoregressive moving average model with exogenous input (ARMAX) verified on a numerical model and then it was used on the data of the mentioned structure and the parameters of the model were estimated using the prediction error method. After that, the natural frequencies of the structure and their corresponding mode shapes were identified. Due to the fact that the data of the mentioned structure was also collected under ambient vibration, frequencies and mode shapes were also identified using the frequency domain decomposition (FDD) method. Comparing the mode shapes from the two methods using the modal confidence matrix showed that the modes of the two methods are similar to each other. But the frequencies detected by ARMAX method were generally lower than the FDD frequencies and this frequency difference was more for the first mode shapes, the slight frequencies discrepancy detected using the two methods can be attributed to special seismic equipment such as BRB braces which are more likely to become active in higher vibration intensities.

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