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Numerical study of the three-span shaft critical speed and develop a new method to compute critical speed

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Abstract

Critical speed is one of the most important characteristic features of rotating machinery. If the running speed of machinery is matched with critical speed, vibration amplitude increases until severe damage results in the machine. By increasing the constraints of the rotor shaft system, the complexity of calculating the critical speed is increased. In this study, a new method based on finite element analysis (FEA) is developed for the computing critical speed of the machinery with four bearings (supports). The modal analysis by FEA is selected to calculate the natural frequency of the shaft. In this regard, FEA is implemented in the experimental setup, and results are compared to verify the validity of the model. The FEA analysis is used for several case studies for different shaft lengths. By validation of the FEA model, it is used to develop a new method for the calculation of the rotor shaft critical system. In the developed method, four equations were derived for four ranks of critical speed. Briefly, the developed method is an algorithm in which the entrance of its mechanical properties of the shaft and its output is four ranks of the critical speed. It will be shown that the first natural frequency of the three-span shaft varies from the fixed-fixed shaft with the length of the middle span to a simple support beam with the length of the middle span shaft. The output of this study helps to calculate the natural frequency of the three-span shaft in which the close solution is complex.

Keywords: shaft critical speed; finite element analysis (FEA); natural frequency; three-span shaft

1. Introduction

Rotordynamics [1, 2] is a branch of the mechanical engineering that deals with the dynamics of rotating machinery such as turbomachinery [3, 4], electrical machines [5], high-speed motors in the oil and gas industry [6], automotive turbochargers [7] and others rotating machinery. One of the most important subjects in rotor dynamics is the critical speed [8, 9] of the shaft, and is one of the most important characteristic features. By matching the running speed of the machine and critical speed, the vibration of the machine increases, so the stability of the machine is an important matter. For avoiding machine instability, the running speed shouldn't equal critical speed [10]. Avoiding resonants in machinery operating condition modal analysis is important in the machinery design [11]. In most machinery, the critical speed is equal to the natural frequency of the machine and in some has a few differences such as journal bearing systems [12]. In this case, the stiffness of the support should be considered.

There are several methods for the calculation of the critical speed. Finite element analysis (FEA) [13] is one of the most practical methods. Wang et al. [14] used 3D finite element analysis to establish the natural frequency and mode shape of the magnetic bearing-rotor system of the high-speed machine. To define the bearing stiffness, they used an exciting test. Their results with the FEA method are consistent with the high-speed permanent magnet method. Huang and Han [15] used the lumped mass method to develop a discrete rotor-shaft assembly. There were large errors in calculating critical speed by the commercial finite element analysis software for their engineering model. Bai et al. [16] analyzed the dynamic characteristic of the main shaft system in Hydro-turbine based on ANSYS software. They used modal analysis and calculate the critical speed of the shafting system. Wang et al. [17] calculated the critical speed of the shaft by a method based on the finite element method and established a rotor-bearing dynamic system using a mixed model of lumped parameter method and distributed mass method, the affecting factors of the gyroscopic moment, supporting stiffness and shear deformation were taken into account. Wang and sun [18] calculate six ranks of the critical speed of turbopump rotor and using a one-dimensional finite element method, considering the mass of shaft, gyroscopic effect, and influence of shearing deformation, established the one-dimensional rotor dynamics, finite element model. In another study, Bai et al. [19] to calculate characteristics of the lateral vibration in the main shaft system of the hydro-turbine generating set proposed a three-node elastic shaft element. They derived dynamic equations and obtained the interpolation function, translational inertia matrix, rotational inertia matrix, gyroscopic matrix, and the stiffness matrix and also developed FEA programming. Jahromi et al. [20] used 3D FEA in the Forward and Backward Whirling of a Rotor with a Gyroscopic effect. Most of the recent studies, focus on one system and didn't generalize their solutions to other rotor-shaft systems.

The present study develops a new method based on the modal analysis by the FEA method. First, a modal analysis of the three-span shaft with the FEA method is developed and the results compare with the experimental setup. By validation of the FEA model, it is used to develop a new method for the calculation of the rotor-shaft critical system. In the developed method, four equations were derived for four ranks of critical speed. The present study assumed that the cross-section of the shaft is similar to the entire rotor-shaft system. Briefly, the developed method is an algorithm in which the entrance of its mechanical properties of the shaft and its output is four ranks of the critical speed. The system is a three-span shaft that has two similar spans. It is shown that with varying the length of side spans from nearly zero to the middle part length, the first natural frequency of the shaft varies from a fixed-fixed beam with the length of the middle span of the system to a simple support beam with the same length. The innovations of this study are easy for implementing, fast performance, and generalization. Its limitation is that the rotor-shaft system has a constant cross-section and has a limitation for side span length.

2. Experimental setup and model

The experimental setup and mathematical model are shown in figure 1. In this setup, four rolling element bearings exist. Because of weak resistance to the moment in support, assume the bearings as simply support. So for the present setup, four simple support exist.

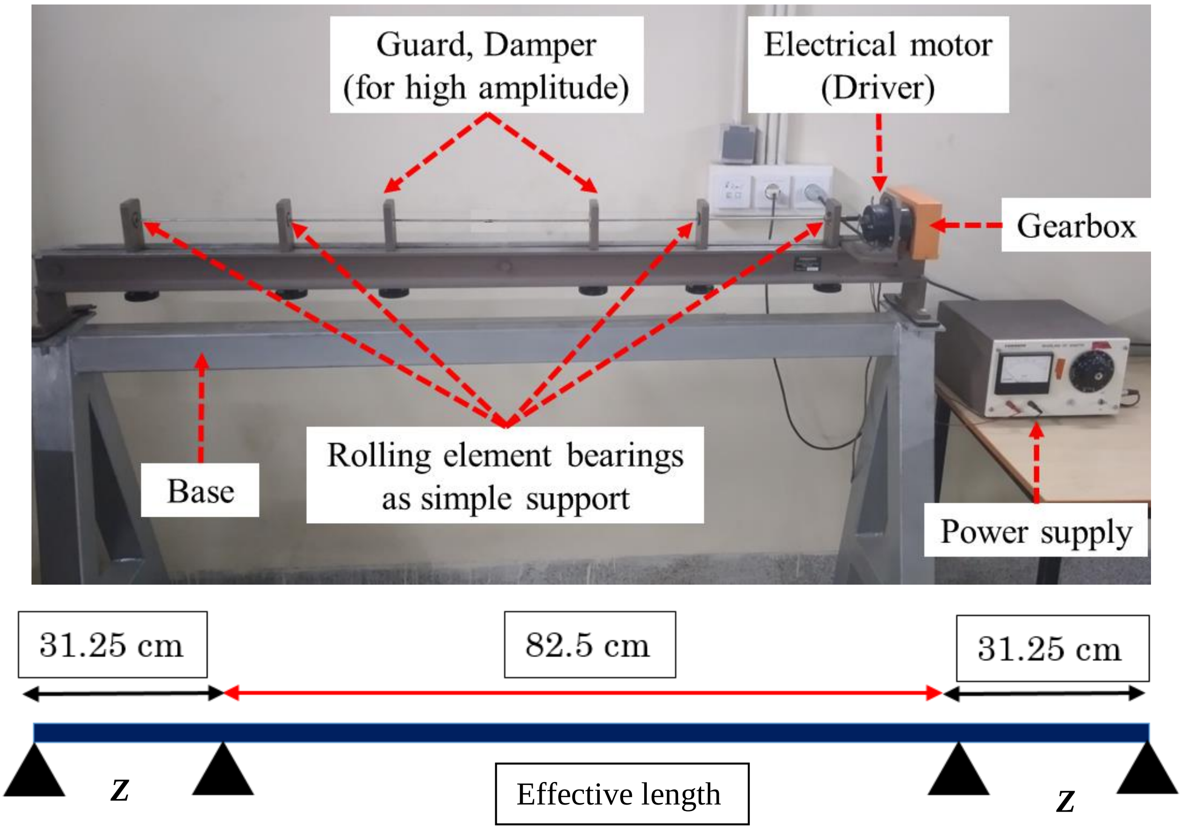

Figure 1. Experimental setup and mathematical model

Table 1, describes the mechanical properties of the shaft.

Table 1. Mechanical properties of the shaft

Shaft Diameter	Young modulus	Mass	Total length	Effective length
4.75 mm	200 GPa	0.12 kg	145 cm	82.5 cm

In table 1, the effective length is also shown in figure 1. With a Power supply, the shaft starts to run, and with a tachometer, the running speed of the shaft is measured. For this setup, the two first critical speeds can be measured. At the running speed which matched with critical speed, the vibration of the shaft increased. With measurement, the two first critical speed is presented in table 2. Variation the speed of the running speed is continuously from starting up to the second critical speed. The setup belongs to the Sharif University of technology vibration lab. Natural frequency and critical speed are different, however, they are assumed to be the same in this study. Because they have a little discrepancy in low-speed machinery cases. Because in low-speed cases, the Gyroscopic effects are negligible.

Table 2. Measured critical speeds (Hz)

First critical speed	Second critical speed
20.8 Hz	62

3. Finite element analysis (Modal) of the beam with 4 simple support

This section describes finite element analysis if the model is described and the results are compared to the experimental method. The model is analyzed in Abaqus CAE and the type of elements assigned to the systems is B31, a 2-node linear beam in space. Figure 2 and table 3, represents the natural frequency of a beam with four simple support. By comparing the results in figure 2 and table 2, obtain that the FEA model is near realistic, has acceptable accuracy, and can be used for the developed model. In this analysis. The mesh size is 0.5 mm and the mesh independence study is done.

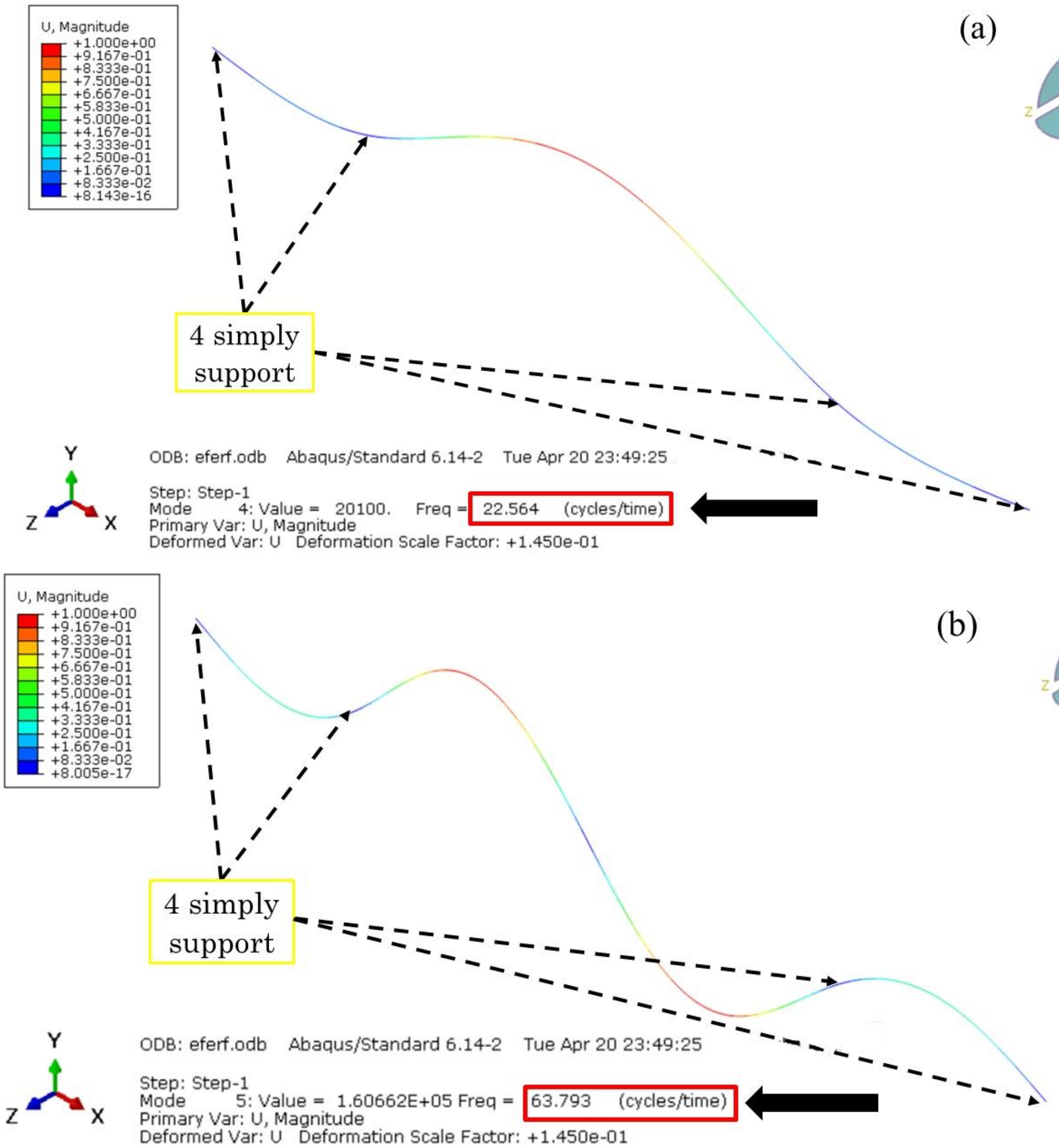

Figure 2. Finite element analysis (Modal analysis) of the beam with four simple support

Table 3. calculated natural frequency (Hz)

First natural frequency	Second natural frequency
22.56 Hz	63.79

In this regard, the analysis can continue with an acceptable FEA model and the constraints and mechanical properties in the FEA model are acceptable.

4. Develop a new method for the determination of four simple support beam natural frequency

4.1 Equivalent the four-simple support beam with fixed-fixed & simple support beam

The analytical solution for fixed-fixed beam and simply supported beam in the modal analysis or free vibration is obtained from equation 1 and just the boundary conditions are different [21]. Because of the free vibration of the system, the right side of the equation is zero.

$$EI(x)\frac{\partial^4 w}{\partial x^4}(x,t) + \rho A(x)\frac{\partial^2 w}{\partial t^2}(x,t) = 0 \tag{1}$$

Where the *E* is shaft young modules and *I* is moment inertia of cross-section *ρ* is shaft density *A* refers to the area of the shaft cross-section and *w(x, t)* refers to shaft deformation in the lateral direction. The binary conditions for the fixed-fixed beam are represented in equation 2.

$$\begin{aligned} w(0,t) &= 0 \\ w(l,t) &= 0 \\ \frac{\partial w}{\partial x}(0,t) &= 0 \\ \frac{\partial w}{\partial x}(l,t) &= 0 \end{aligned} \tag{2}$$

The final solution for natural frequency for the fixed-fixed beam is presented in equation 3.

$$\begin{gathered} \cos \beta l \cosh \beta l = 1 \\ \beta_n l \cong \frac{2n+1}{2}\pi \\ \omega_n = (\beta_n l)^2 \sqrt{\frac{EI}{\rho A l^4}} \end{gathered} \tag{3}$$

For effective length and parameters were defined in table 1, and the natural frequency of the fixed-fixed beam is defined in table 4. The *l* is equal to the effective length. Now by variation of the *Z* length in figure 1, from 0.3125 m to zero, the natural frequency of the system is computed. At *Z*=0 the system is equivalent to a fixed-fixed beam. The *Z*=0 is equal to the state that two bearings are very close together, control two close points, and perform as fixed support. Therefore the rotation is close to zero. When *Z* is equal to effective, the beam consists of three similar sections, so the first natural frequency of the system is equivalent to a simple support beam (figure 3). It is understood that the natural frequency of the system changes from the first natural frequency of a simple support beam to a fixed-fixed beam by variation *Z* from effective length to nearly zero. The mode shape of the aforementioned states is shown in figure 3. When the Z parameter length is equal to the effective length, the three-span shaft, consists of three same spans, so the first natural frequency of the systems is equal to the simple support shaft. The mode shape, in this case, is such as the sine function.

Table 4. The computed natural frequency of fixed-fixed

Parameter	value	Parameter	value
$\beta_1 l$	4.712	$\beta_3 l$	10.9956
$\beta_2 l$	7.854	$\beta_4 l$	14.1372
The natural frequency of the first mode	30.433 Hz	The natural frequency of the third mode	165.71 Hz
The natural frequency of the second mode	84.537 Hz	The natural frequency of the fourth mode	273.94

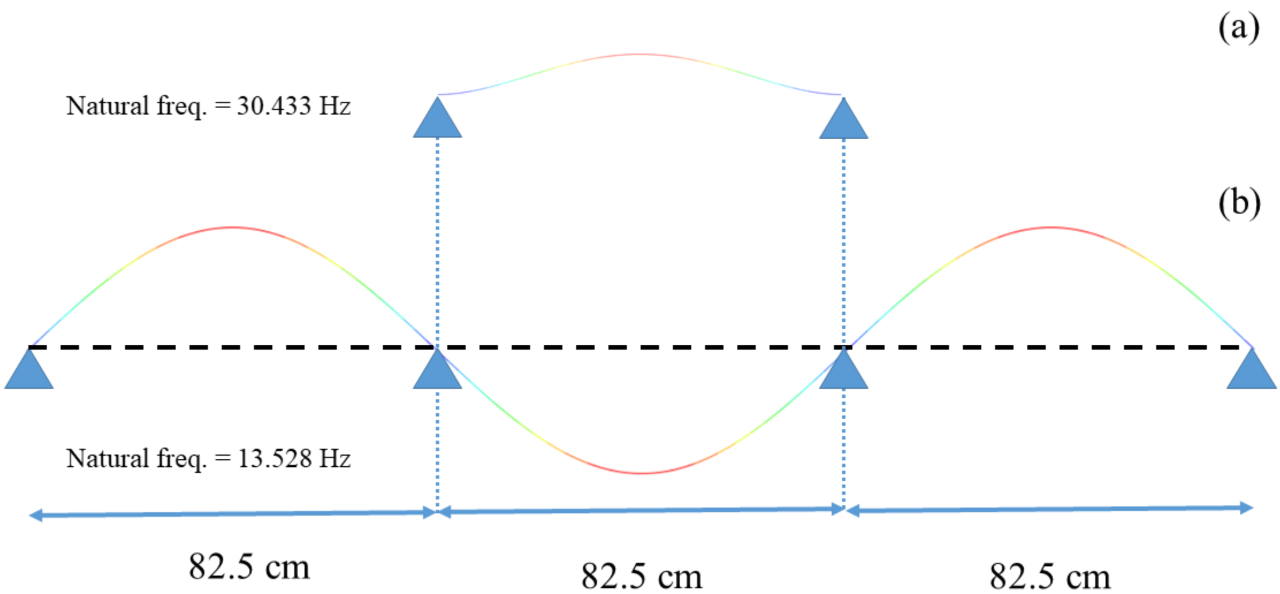

Figure 3. Mode shapes of the three-span beam by variation Z length from nearly zero (a) to effective length (b). The first natural frequency is also changed between the two cases.

By increasing the Z length, the natural frequencies of the system decreased. The variation of the first four natural frequencies with Z is represented in figure 4. In figure 4 and all the next figures, the resolution of the Z parameter is 1 cm for precise analysis.

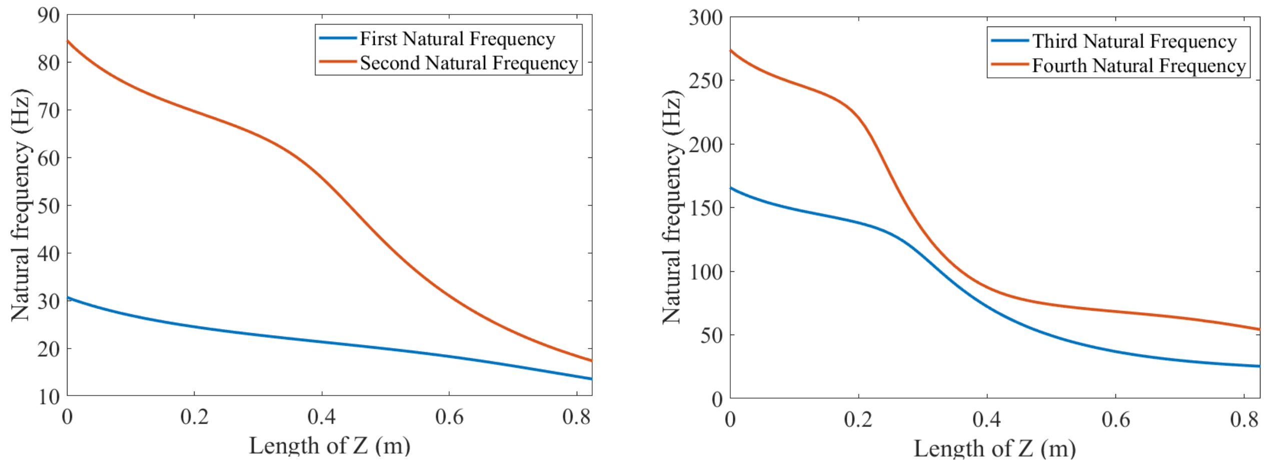

Figure 4. The first four natural frequencies of the three-span beam in the present study

4.2 Develop a new method for computing a three-span beam's natural frequency

By finite element analysis, the equations which describe the natural frequency of a three-span shaft in terms of the effective length and *Z* parameter are extracted. Figure 5, represents the variation of the natural frequency of the shaft with effective length and *Z* parameter. Fitting an equation to the curve of the natural frequencies, earn four functions to compute the natural frequencies of the beam. So a general state should be selected for the fitting curve. The effective length equal to 1 m is selected for the general state because in this case performs as a unit value. It is clear that the equation of the fitting curve in figure 5, is similar to the unit. The aim is to find equations for computing the first four natural frequencies. Figure 6, represents the process of the proposed method. The equations used in figure 6, are represented in table 5. The range for these equations is described in figure 7. All curve has a similar general shape. This method is similar to dimensional analysis.

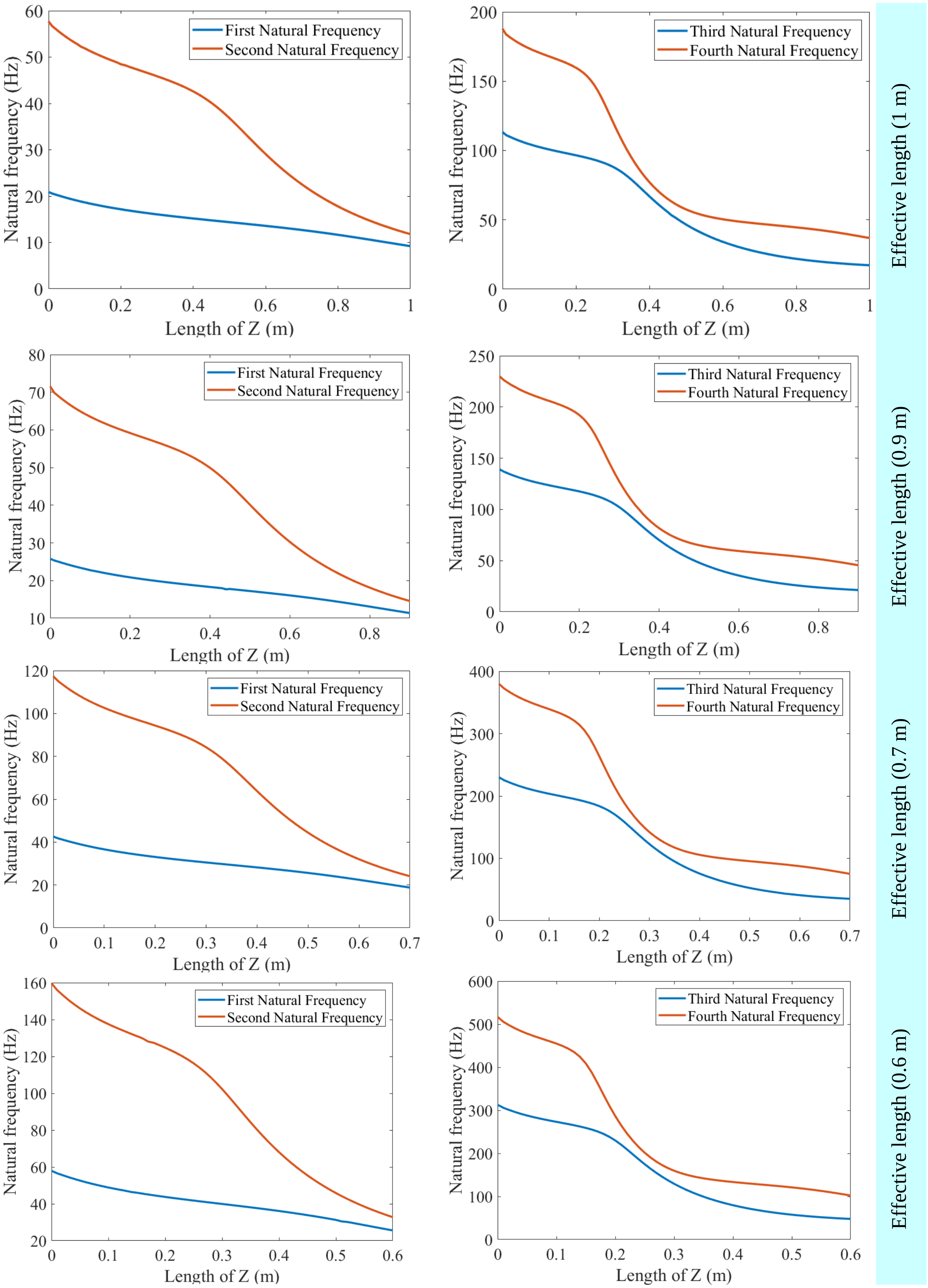

Figure 5. The first four natural frequencies of the three-span beam for several effective lengths

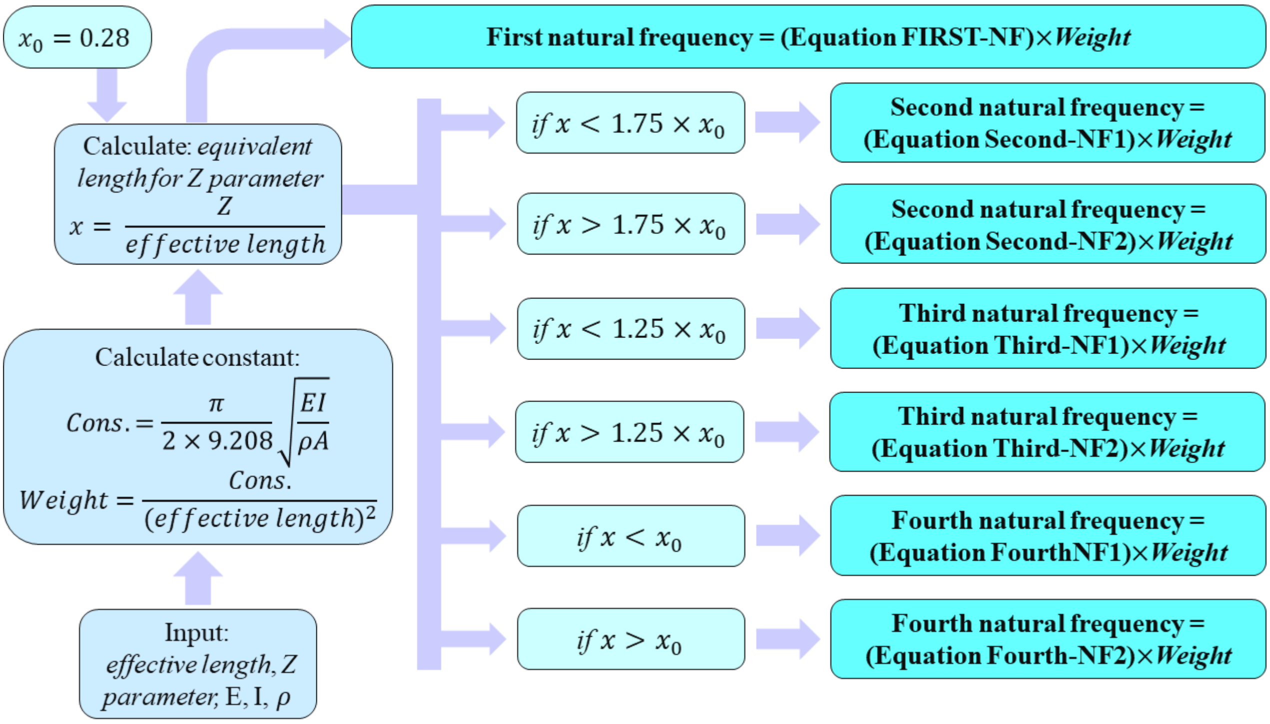

Figure 6. The proposed method for calculating four ranks of three-span shaft critical speed

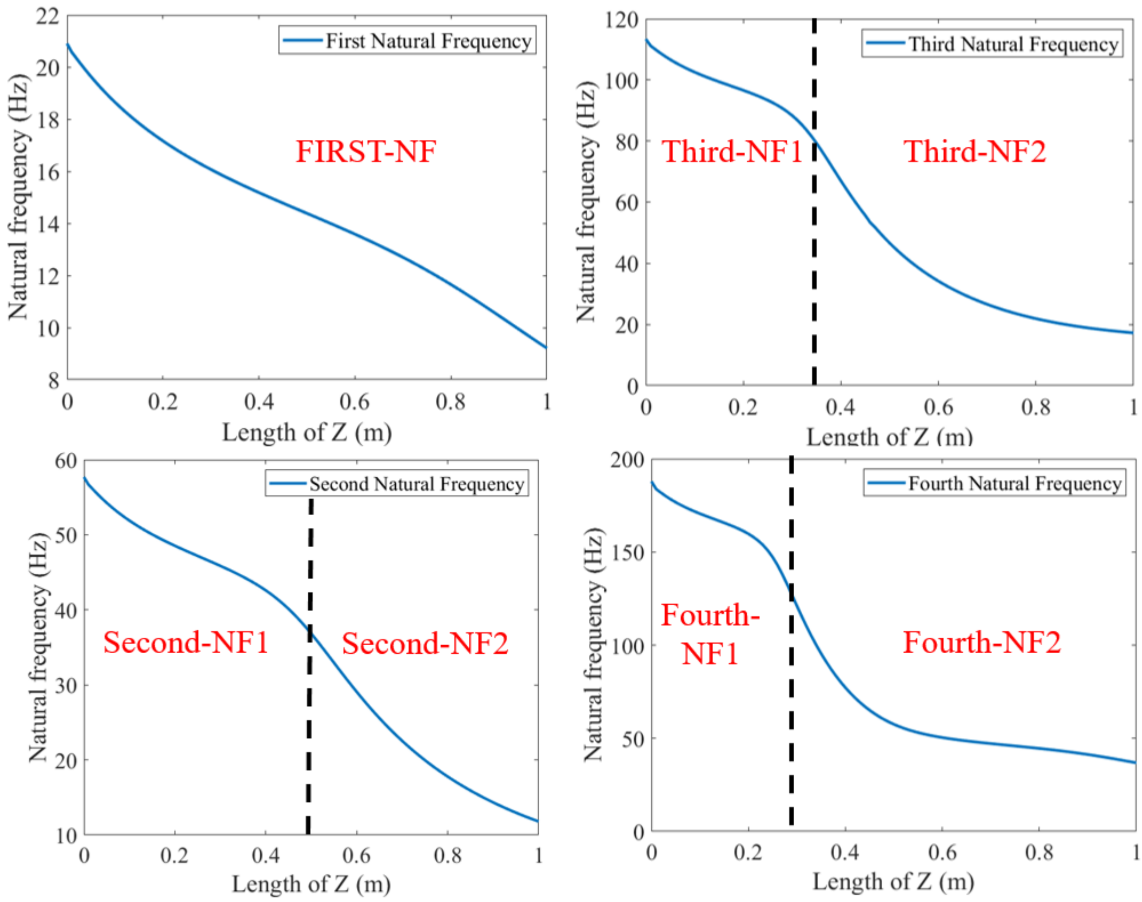

Figure7. The intervals were described in the curve fitting

Table 5. The equation used for the proposed method for calculating four ranks of three-span shaft critical speed

Name	Case study	Equation
FIRST-NF	Detail described in table 1	$-15.29x^3 + 24.87x^2 - 21.21x + 20.61$
Second-NF1		$-250.6x^3 + 187.7x^2 - 72.38x + 57.49$
Second-NF2		$71.28x^2 - 156.5x + 97.29$
Third-NF1		$-1811x^4 - 143.6x^3 + 401.9x^2 - 140.7x + 112.8$
Third-NF2		$-286.2x^3 + 772.3x^2 - 718x + 248.7$
Fourth-NF1		$-1.31e^4x^4 + 1.84e^4x^3 + 670x^2 - 237x + 187$
Fourth-NF2		$1318x^4 - 4195x^3 + 4931x^2 - 2564x + 549.1$

4.3 Another method for calculating the first natural frequency

Another method in literature [22] for calculating the critical speed and natural frequency is presented in equation 4.

$$\omega_{cr} = \sqrt{\frac{Kg}{W}} \tag{4}$$

Where K is system equivalent stiffness and W is equivalent weight and g is gravity acceleration. It is possible to convert equation 4 to equation 5.

$$\omega_{cr} = \sqrt{\frac{g}{\delta}} \tag{5}$$

Where the δ is equivalent deflection. In this regard, the maximum deflection of the three-span shaft is calculated by FEA. Equation 6 represents the beam element in finite element analysis [23]. The static general analysis is selected in ABAQUS CAE and also MATLAB programming of the beam element. The results (figure 8) show that there is little difference in calculated critical speed between the two methods. The results of the FEA are more precise.

$$\mathbf{k} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \tag{6}$$

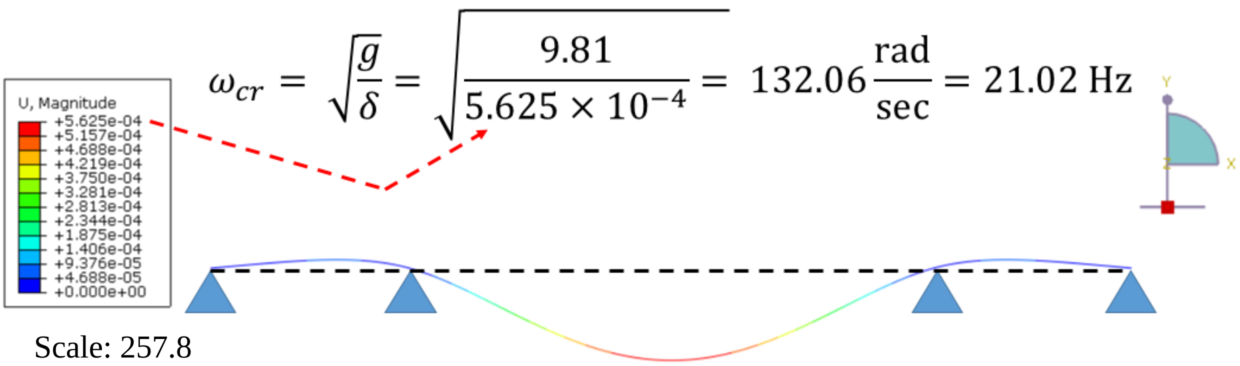

Figure 8. First critical speed calculation with gravity and deflection method

4.4 Results and discussion

The proposed method which was described in figure 6, is implemented in other cases and the results were noticeable and the accuracy was 100 % whereas this algorithm has limitations. It is responsible for the (0 1] interval for the *Z* parameter. whereas this is practical for the industry because the middle span has a higher length. In this algorithm, it is possible to choose every length for the middle span, whereas the side spans should be shorter than the middle span. By calculating the equivalent length for the *Z* parameter the algorithm works very well. For example, in figure 5 it is possible to calculate the equivalent length for the *Z* parameter for all aforementioned states. Briefly, this algorithm works for equivalent length for the *Z* parameter in (0 1] intervals or the ratio of the Z parameter to effective length is equal to or less than 1. Table 6 describe some equivalent length for the *Z* parameter.

Table 6. Some examples for calculating equivalent length for the *Z* parameter

Effective length	Z parameter	Equivalent length for Z parameter
0.9	0.36	0.4
0.7	0.14	0.2
0.6	0.3	0.5
0.5	0.1	0.2
0.825	0.495	0.6

5. Conclusion

In this study, a new method based on the finite element method was developed for calculating the critical speed of the three-span shaft. Three-span shaft has two similar side spans. It was shown that the first natural frequency of the three-span shaft varies from the fixed-fixed shaft with the length of the middle span to a simple support beam with the length of the middle span shaft. The difference between the results of the experimental and FEA method is about 7% which is negligible. The method consists of polynomial equations which were fitted to the curve of the FEA method results for modal analysis. The innovations of this study are easy for implementing, fast performance, and generalization. Its limitation is that the rotor-shaft system has a constant cross-section and has a limitation for side span length. The results were also compared with other theoretical methods and experiments that confirm this method and its closing to reality.

REFERENCES

1. Muszynska, A., *Rotordynamics*. 2005: CRC press.
2. Vance, J.M., F.Y. Zeidan, and B.G. Murphy, *Machinery vibration and rotor dynamics*. 2010: John Wiley & Sons.
3. Childs, D., *Turbomachinery rotordynamics: phenomena, modeling, and analysis*. 1993: John Wiley & Sons.
4. Vance, J.M., *Rotordynamics of turbomachinery*. 1991: John Wiley & Sons.
5. Huang, Z. and Y. Le, *Rotordynamics modeling and analysis of high-speed permanent magnet electrical machine rotors.* IET Electric Power Applications, 2018. **12**(8): p. 1104-1109.
6. Singhal, S. *Vibration and rotor dynamics of large high-speed motors driving compressors in the oil and gas industry*. in *2012 Petroleum and Chemical Industry Conference (PCIC)*. 2012. IEEE.
7. Nguyen-Schäfer, H., *Rotordynamics of automotive turbochargers*. 2015: Springer.
8. O¨ zgu¨ ven, H.N., *On the critical speed of continuous shaft-disk systems.* 1984.
9. Swanson, E., C.D. Powell, and S. Weissman, *A practical review of rotating machinery critical speeds and modes.* Sound and vibration, 2005. **39**(5): p. 16-17.
10. Zajaeczkowski, J., *Stability of rotation of a motor-driven shaft above the critical speed.* Journal of Sound and Vibration, 1998. **209**(5): p. 857-865.
11. Jalali, M.H., et al., *Dynamic analysis of a high-speed rotor-bearing system.* Measurement, 2014. **53**: p. 1-9.
12. Someya, T., et al., *Journal-bearing databook*. 2013: Springer Science & Business Media.
13. Ramsey, K., *Experimental modal analysis, structural modifications and FEA analysis on a desktop computer.* SOUND AND VIBRAT., 1983. **17**(2): p. 19-27.
14. Wang, T., et al. *Stiffness and critical speed calculation of magnetic bearing-rotor system based on FEA.* in *2008 International Conference on Electrical Machines and Systems*. 2008. IEEE.
15. Huang, Z. and B. Han, *Effective approach for calculating critical speeds of high-speed permanent magnet motor rotor-shaft assemblies.* IET Electric Power Applications, 2015. **9**(9): p. 628-633.
16. Bai, B., et al., *Analysis of dynamic characteristics of the main shaft system in a hydro-turbine based on ANSYS.* Procedia Engineering, 2012. **31**: p. 654-658.
17. Wang, J.F. and K. Sun. *Rotor critical speed calculation based on finite element method of a mixed model*. in *Advanced Materials Research*. 2012. Trans Tech Publ.
18. Wang, J.F. and K. Sun. *The calculation of the rotor critical speed of turbopump*. in *Advanced Materials Research*. 2012. Trans Tech Publ.
19. Bai, B. and L.X. Zhang. *A 3-node shaft element for main shaft vibration FE analysis*. in *Applied Mechanics and Materials*. 2014. Trans Tech Publ.
20. Jahromi, A.F., R.B. Bhat, and W.-F. Xie, *Forward and backward whirling of a rotor with gyroscopic effect*, in *Vibration Engineering and Technology of Machinery*. 2015, Springer. p. 879-887.
21. Rao, S.S., *Vibration of continuous systems*. 2019: John Wiley & Sons.
22. Gunter, E., *Introduction to Rotor Dynamics, Critical speed and unbalance response analysis*. 2001, RODYN Vibration Analysis, Inc. Charlottesville, VA.
23. Filippa, C.A., *Introduction to finite element methods.* University of Colorado, 2004. **885**.