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Free longitudinal vibrations analysis of nanorods based on Eringen's nonlocal theory with considering surface energy using wave approach

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Abstract

In this paper analysis of free longitudinal vibrations of continuous nanorod (nanotube) in cylindrical coordinate system, have been done by wave method. Therefore, first the axial force equation of nanorod is derived from its motion equation. Then by using of Eringen's nonlocal elasticity theory, nonlocal axial force of nanorod is obtained. After deriving axial force equation, which includes surface energy parameters, equations of motion and axial force of nanorod in terms of positive-going and negative-going waves are obtained. By placing the spring at the end of the boundaries of the nanorod, wave propagation matrix is obtained under boundary conditions of clamped-clamped and clamped-free. Eventually by calculation the dimensionless frequency of nanorod for both of the mentioned boundary conditions, dimensionless frequency changes vs nonlocal parameter and also vs surface energy parameters as diagram have been investigated. In each diagram, first three modes of dimensionless natural frequency under difference poisson's ratios are investigated.

Keywords: longitudinal vibration; nanorod; Eringen's theory; surface energy.

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1. Introduction

After discovery of carbon nanotubes (CNT's) by Ijima [1] and also, with the rapid development of science and technology, the use of nanotubes has been expanded in many industrial applications. Carbon nanotubes are receiving a lot of attention due to their unique mechanical, electrical, chemical and thermal properties. Nanorods (nanotubes) studies have attracted the attention of many researchers. Harris [2] researched structure of carbon nanotubes. Yan et al [3] analysed the forced vibration and strength of carbon nanotubes. Aydogdu [4] analysed the longitudinal wave propagation of multiple walls carbon nanotubes. Loghmani et al [5] analysed the longitudinal vibration of nanorods with discontinuity based on nonlocal elasticity theory by wave approach. Arash and Ansari [6] evaluated the effect of the nonlocal parameter in the vibrations of the single-wall carbon nanotube with initial strain. Kiani [7] investigated the longitudinal free vibrations of the conical nanowire based on nonlocal elasticity theory. Danesh et al [8] analysed the axial vibrations of the tapered nonorod based on nonlocal elasticity theory and differential quadrature method. Murmu and Adhikari [9] investigated about nonlocal effects in the longitudinal vibration of double-nanorod systems. Chang et al [10] analysed vibration of viscoelastic carbon nanotubes. Reddy [11] investigated about nonlocal theories for buckling, bending and vibration of beams. Loya et al [12] analysed free transverse vibrations of cracked nanobeam's by using a nonlocal elasticity model. In this paper, we intend to analyse the movement of nanorods (CNTs) by wave method and analyse its behaviour in dimensionless conditions.

2. Governing equations on nanorods

Consider a nanorod (nanotube) with inner radius R_1 and outer radius R_2 , as shown in fig.1 :

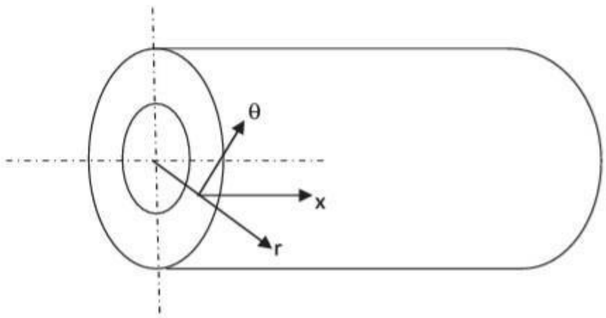

Figure 1. Geometry of CNT and cylindrical coordinate [13]

Based on Hamilton's principle and Huke's law, the longitudinal motion equation of nanotube has been defined [13]:

$$\rho S\frac{\partial^2 u}{\partial t^2}-ES\frac{\partial^2 u}{\partial x^2}+\frac{2\upsilon(\gamma_1 R_2+\gamma_2 R_1)S}{R_1 R_2}\frac{\partial^2 u}{\partial x^2}-\rho\upsilon^2 J\frac{\partial^4 u}{\partial t^2\partial x^2}+G\upsilon^2 J\frac{\partial^4 u}{\partial x^4}=0 \tag{1}$$

In which ρ is density and S is cross section of nanorod and υ is poisson's ratio and J is polar inertia moment and G,E are shear and young elasticity modulus respectively.

By using of Eringen's elasticity theory [14] Eq. (2), we have nonlocal motion equation of nanorod, Eq. (3):

$$[1-(e_0 a)^2\nabla^2]\tau_{kl}=\sigma_{kl} \tag{2}$$

$$\rho S\frac{\partial^2 u}{\partial t^2}-ES\frac{\partial^2 u}{\partial x^2}-\rho \upsilon^2 J\frac{\partial^4 u}{\partial x^2\partial t^2}+\upsilon^2 GJ\frac{\partial^4 u}{\partial x^4}-(e_0a)^2\rho S\frac{\partial^4 u}{\partial t^2\partial x^2}+(e_0a)^2\upsilon^2 J\rho\frac{\partial^6 u}{\partial x^4\partial t^2}+\frac{2\upsilon(\gamma_1R_2+\gamma_2R_1)S}{R_1R_2}\frac{\partial^2 u}{\partial x^2}=0 \quad (3)$$

3. Axial force of nanorods

As we know the axial force of nanorods will be obtained in local, from Eq. (4) and in nonlocal from Eq. (5) :

$$N_{xx}{}^{L}=\int_S \sigma_{xx}ds \quad (4)$$

$$N_{xx}^{NL}=\int_S \tau_{xx}ds \quad (5)$$

in which σ_{xx} is normal stress and τ_{xx} is shear stress.

To obtain the axial force equation of nanorod in local, first should write Eq. (2) as follows:

$$\tau_{xx}-(e_0a)^2\frac{\partial^2\tau_{xx}}{\partial x^2}=E\frac{\partial u(x.t)}{\partial x} \quad (6)$$

in which $\nabla^2$is the Laplace operator that is define as :

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial x^2} \quad (7)$$

integrating Eq. (6) with respect to cross section gives the following equation:

$$N_{xx}^{NL}-(e_0a)^2\frac{\partial^2 N_{xx}^{NL}}{\partial x^2}=ES\frac{\partial u(x,t)}{\partial x} \quad (8)$$

Through Eq. (8), the axial force equation can be obtained. Eqs. (4) and (5) yield to:

$$\frac{\partial N_{xx}^{NL}}{\partial x}=\rho S\frac{\partial^2 u(x,t)}{\partial t^2} \quad (9)$$

$$\frac{\partial N_{xx}^{L}}{\partial x}=ES\frac{\partial^2 u(x,t)}{\partial x^2} \quad (10)$$

substituting Eqs. (9) and (10) in Eq. (8) , we have:

$$\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right)\rho S\frac{\partial^2 u}{\partial t^2}=ES\frac{\partial^2 u}{\partial x^2} \quad (11)$$

considering the nanorod as a nanotube with inner and outer radiuses and the surface energy parameters, we can write the following equation:

$$\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right)\left(\rho \upsilon^2 J\frac{\partial^2}{\partial x^2}-\rho S\right)\frac{\partial^2 u}{\partial t^2}=\left(\frac{2\upsilon(\gamma_1R_2+\gamma_2R_1)S}{R_1R_2}-ES+G\upsilon^2J\frac{\partial^2}{\partial x^2}\right)\frac{\partial u^2}{\partial x^2} \quad (12)$$

Which this equation is a same equation of free longitudinal vibration of nanorod in nonlocal by considering the surface energy parameters.

Now by using of Eqs. (6) and (12), the following equation can be obtained :

$$\frac{\partial N_{xx}^{NL}}{\partial x}=\left(\rho\upsilon^2J\frac{\partial^2}{\partial x^2}-\rho S\right)\frac{\partial^2 u}{\partial t^2} \quad (13)$$

By deriving Eq. (13) with respect to x, we have:

$$\frac{\partial^2 N_{xx}^{NL}}{\partial x^2}=\rho\upsilon^2J\frac{\partial^5 u}{\partial x^3\partial t^2}-\rho S\frac{\partial^3 u}{\partial t^2\partial x} \quad (14)$$

By substituting Eq. (14) in Eq. (8) the nonlocal axial force equation of nanotube can be written as:

$$N_{xx}^{NL}(x.t)=\left[ES+(e_0a)^2\left(\rho\upsilon^2J\frac{\partial^4}{\partial x^2\partial t^2}-\rho S\frac{\partial^2}{\partial t^2}\right)\right]\frac{\partial u}{\partial x} \quad (15)$$

4. Analysis by using wave approach

Using the method of separation of variables, we can write the axial motion equation of nanorod as follows [5]:

$$u(x.t)=[A_1e^{-ikx}+A_2e^{ikx}]e^{i\omega t}=[A_1e^{-i(kx-\omega t)}+A_2e^{i(kx+\omega t)}] \quad (16)$$

In which, A_1 and A_2 are constants, k and ω are wave number and natural frequency respectively. The equation (16) can be expressed as positive-going a^+ and negative-going a^- waves:

$$u(x.t)=(a^+ + a^-)e^{i\omega t} \quad . \quad \{a^+=A_1e^{-ikx}\ \ a^-=A_2e^{ikx}\} \quad (17)$$

Substituting Eq. (16) in Eq. (4), the relationship between natural frequency and wave number for nanorod (CNT) in nonlocal elasticity theory is obtained as follows:

$$k=\left[\frac{\rho S\omega^2(e_0a)^2+\rho\upsilon^2J\omega^2+\frac{2\upsilon(\gamma_1R_2+\gamma_2R_1)S}{R_1R_2}-ES\pm\left[\left(ES-\rho S\omega^2(e_0a)^2-\frac{2\upsilon(\gamma_1R_2+\gamma_2R_1)S}{R_1R_2}-\rho\upsilon^2J\omega^2\right)^2+4\left(\upsilon^2GJ-\rho\upsilon^2J(e_0a)^2\omega^2\right)\left(\rho S\omega^2\right)\right]^{1/2}}{2\upsilon^2GJ-2\upsilon^2J(e_0a)^2\omega^2}\right]^{1/2} \quad (18)$$

By deriving Eq. (16) and substitution it in Eq. (15) and considering Eq. (17), the axial force equation of the nanorod can be express in terms of the positive-going and negative-going waves as:

$$N(x.t) = [-ikES + i(e_0a)^2(-\rho v^2 J\omega^2 k^3 - \rho S\omega^2 k)](a^+ - a^-)e^{i\omega t} \tag{19}$$

4.1 Nanorod boundary

We use the axial spring with stiffness $\bar{K}_T$, to model boundary conditions of the nanorod, actually axial force of the nanorod is the same force that opposes displacement of the spring at its boundary [5], so :

$$N(x.t) = -\bar{K}_T u(x.t) \tag{20}$$

By substituting Eqs. (17) and (19) in Eq. (20), we have:

$$[-ikES + i(e_0a)^2(-\rho v^2 J\omega^2 k^3 - \rho Sk\omega^2)](a^+ - a^-) = -\bar{K}_T(a^+ + a^-) \tag{21}$$

Tithe following relationship exists between incident wave and reflected wave at the boundary:

$$a^- = r_g a^+ \tag{22}$$

in which r_g is the reflection function for general spring boundary and it can be written as:

$$r_g = \left[\frac{1+iK_T}{1-iK_T}\right] \tag{23}$$

where, K_T is dimensionless stiffness for spring boundary.

If at the boundary of nanorod $\bar{K}_T \to \infty$, the boundary is clamped and if $\bar{K}_T \to 0$ it's free, so:

$$r_{cl} = [-1] \quad . \quad r_{fr} = [1] \tag{24}$$

In which, subscripts cl and fr are clamped and free boundary conditions, respectively.

4.2 Analysis boundary conditions of nanorod

As shown in figure 2, two boundaries (A and B) exists at the ends of the nanorod. Therefore, $a^{\mp}$ and $b^{\pm}$ are incident and reflected waves at the boundaries A and B, respectively. So we can write:

$$b^+ = F(L)a^+ \quad . \quad a^- = F(L)b^- \tag{25}$$

in which $F(L)$, is propagation function and in continuous nanorods, it's equal for positive and negative directions :

$$F^+(x) = F^-(x) = F(x) = [e^{-ikx}] \tag{26}$$

By considering Eq. (22), reflection relations for the boundaries are as follows:

$$a^{+} = r_A a^{-} \quad . \qquad b^{-} = r_B b^{+} \tag{27}$$

so, Eqs. (26) and (28) can be written in matrix form as :

$$\begin{bmatrix} -1 & r_A & 0 & 0 \\ F(L) & 0 & -1 & 0 \\ 0 & -1 & 0 & F(L) \\ 0 & 0 & r_B & -1 \end{bmatrix} \begin{Bmatrix} a^{+} \\ a^{-} \\ b^{+} \\ b^{-} \end{Bmatrix} = 0 \tag{28}$$

By solving above equation, the natural frequencies of continuous nanorod under any boundary conditions will be obtained as:

$$F(L)^2 r_A r_B - 1 = 0 \tag{29}$$

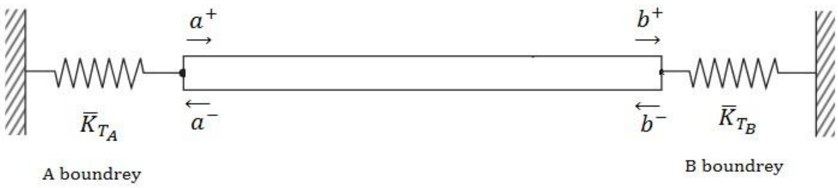

Figure 2. Continuous nanorod with spring at the boundaries

4.2.1 Clamped-free boundary condition

For this boundary condition, reflection matrices at the boundaries A and B are $\boldsymbol{r_A} = [-\mathbf{1}]$ and $\boldsymbol{r_B} = [\mathbf{1}]$. By substituting these values in Eq. (29) and considering Eq. (27), we have:

$$\boldsymbol{e^{-2ikL} + 1 = 0} \tag{30}$$

By solving Eq. (30) we have an exact closed-form solution for clamped-free boundary as follows [5] :

$$kL = \frac{(2n-1)}{2}\pi \qquad . \ n = 1.2.3. \ldots \tag{31}$$

Natural frequency of nanorods will be obtained from following equation:

$$\Omega = \omega L \sqrt{\frac{\rho}{E}} \tag{32}$$

Therefore by substituting Eq. (16) in Eq. (4) ω, and then by substituting it in Eq. (32), dimensionless frequencies equation of the nanorod vs nonlocal parameter ($^{e_0 a}/_{L}$) for clamped-free boundary condition will be obtained :

$$\Omega_n = (2n-1)\frac{\pi}{2}\sqrt{\frac{ES+Gv^2J(\frac{(2n-1)\pi}{2L})^2-\frac{2v(\gamma_1R_2+\gamma_2R_1)S}{R_1R_2}}{ES+Ev^2J(\frac{(2n-1)\pi}{2L})^2+(\frac{e_0a}{L})^2[ES\left(\frac{(2n-1)\pi}{2}\right)^2+Ev^2J\frac{((2n-1)\pi)^4}{16L^2}]}} \qquad (33)$$

4.2.2 Clamped-Clamped boundary condition

For this boundary condition, reflection matrices are equal and both of them are $r_A = r_B = [-1]$. By substituting them in Eq. (29) and considering Eq. (26), we have:

$$e^{-2ikL} - 1 = 0 \qquad (34)$$

By solving above equation, we have an exact closed-form solution for clamped-clamped boundary as follows [5]:

$$kL = n\pi \quad . \quad n = 1.2.3. \dots \qquad (35)$$

Therefore by substitution Eq. (16) in Eq. (4) ω, and then by substituting it in Eq. (32), dimensionless frequencies equation of the nanorod vs nonlocal parameter ($^{e_0a}/_L$) will be obtained :

$$\Omega_n = n\pi\sqrt{\frac{ES+v^2GJ(\frac{n\pi}{L})^2-\frac{2v(\gamma_1R_2+\gamma_2R_1)S}{R_1R_2}}{ES+Ev^2J(\frac{n\pi}{L})^2+(\frac{e_0a}{L})^2[ES(n\pi)^2+Ev^2J\frac{(n\pi)^4}{L^2}]}} \qquad (36)$$

5. Conclusions

In this part, we are going to analyse dimensionless frequency variations vs parameters $(\gamma_1.\gamma_2.\frac{e_0a}{L})$.

5.1 Investigating dimensionless frequency of uniform nanorods vs nonlocal parameter

Figures 3 and 4 show first three dimensionless frequencies vs nonlocal parameter for clamped-free and clamped-clamped boundary conditions, respectively. There are nine graph in the both of them, which represent first three frequency modes under three different poisson's ratio. Actually, we can see that, increasing the nonlocal parameter has decreased the frequency. Also we can see, that increasing of poisson's ratio in each step, will cause a slight decrease in dimensionless frequency.

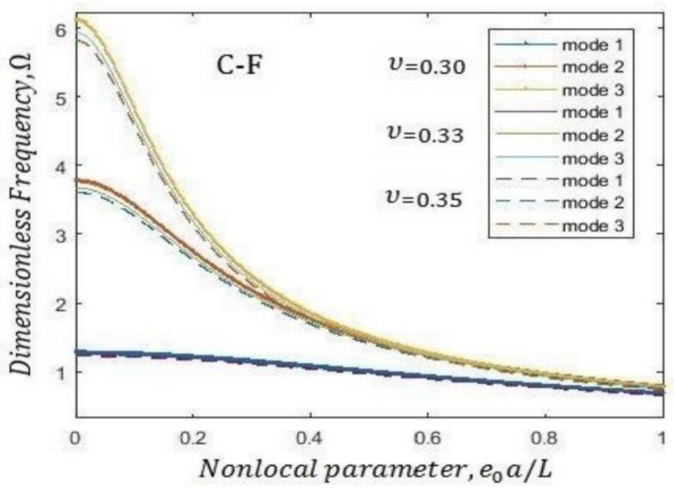

Figure 3. Dimensionless frequency vs nonlocal parameter for clamped-free boundary

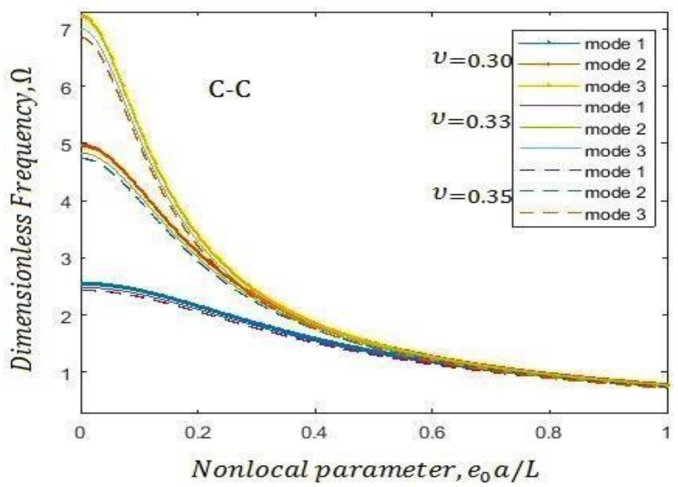

Figure 4. Dimensionless frequency vs nonlocal parameter for clamped-clamped boundary

5.2 Investigating dimensionless frequency of uniform nanorods vs the first surface energy parameter

Figure 5 and 6 show first three dimensionless frequency vs γ_1, for clamped-free and clampedclamped boundary conditions. In both diagrams, by increasing surface energy parameter, the dimensionless frequency has decreased and this process has happen in all the three steps with different poisson's ratios. Also by increasing poisson's ratio in each steps, dimensionless frequencies have been decreased.

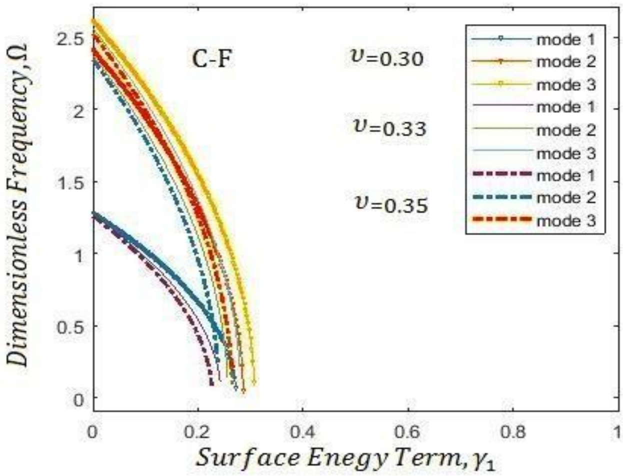

Figure 5. Dimensionless frequency vs first surface energy parameter for clamped-free boundary

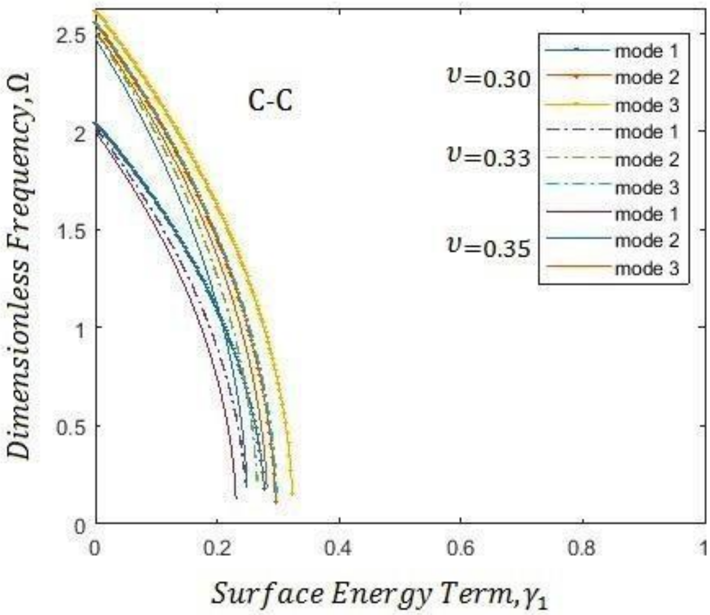

Figure 6. Dimensionless frequency vs first surface energy parameter for clamped-clamped boundary

5.3 Investigating dimensionless frequency of uniform nanorods vs the second surface energy parameter

Figures 7 and 8 show first three dimensionless frequency vs γ_2, for clamped-free and clamped-clamped boundary conditions. We can see in both of the diagrams, for each three steps, by increasing surface energy parameter, dimensionless frequencies have been decreased. In addition, for both of the boundary conditions, by increasing the poisson's ratio in each three steps, the dimensionless frequencies have been decreased.

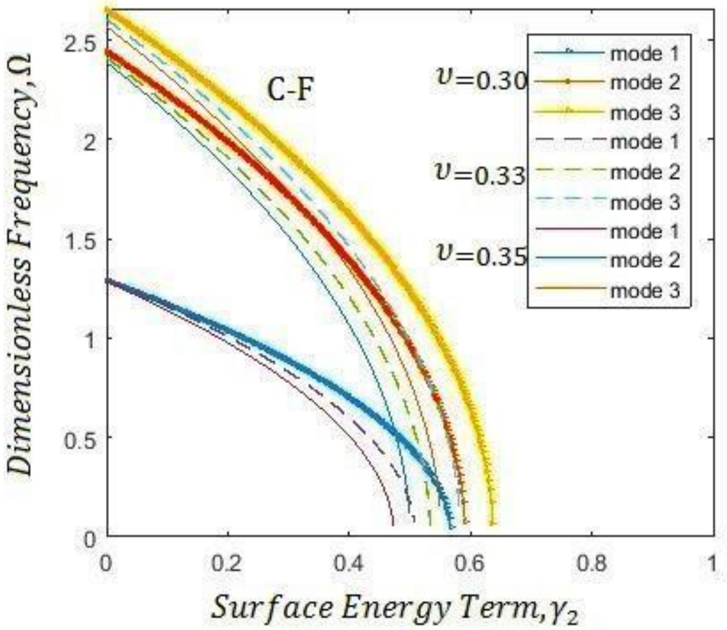

Figure 7. Dimensionless frequency vs second surface energy parameter for clamped-free boundary

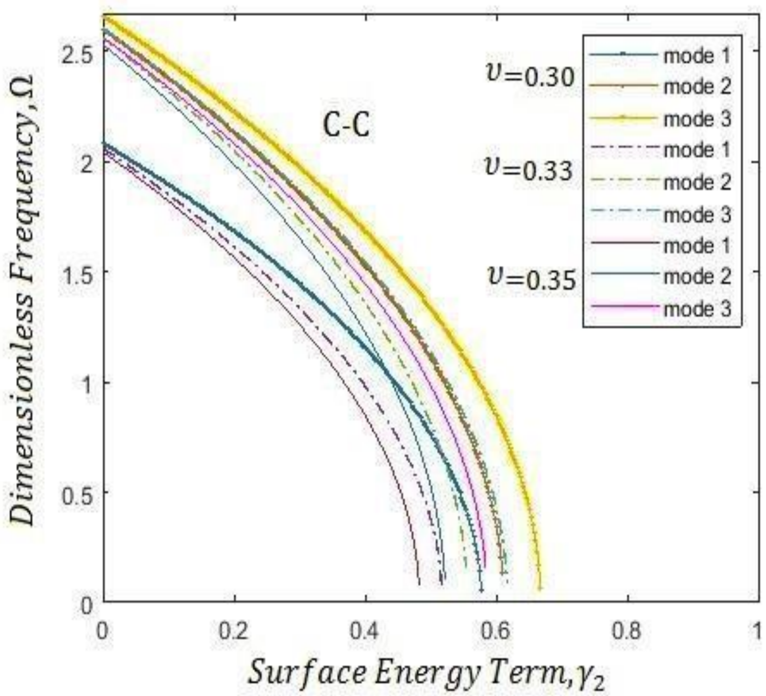

Figure 8. Dimensionless frequency vs second surface energy parameter for clamped-clamped boundary

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